# Philosophizing with Children in the Course of Solving Modeling Problems in a Sixth Grade Mathematics Classroom

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**Introduction**

While the concept of a community of inquiry based on dialogue is an integral part of philosophy for children (Lipman 1988, 2003; Morehouse 2010), this concept is less prevalent in mathematics and science classes. In these subjects emphasis is usually placed on transmitting factual information as accurately and completely as possible. Children expect the teacher (or some other authority) to tell them what the "right" answer to a question is, and teachers expect children to reproduce that answer. There is little opportunity for uncertainty, query and dialog (see Gallas 1995, pp. 7-16; Sprod 2011, p. xiv). Discussions are often conducted between individual students and the teacher, but not among the students themselves. This style of teaching generates a kind of pressure that many children find onerous (Hausberg 2007). In addition, it encourages a vision of mathematics and science as fields of study that are highly abstract, detached from reality, and unfathomable. Pupils leave the classroom thinking that there is a predetermined right and wrong answer for everything in these subjects and no room for discovery. Therefore proponents of constructivist science education (e.g. Driver 1994) and philosophizing in science classes (see Nevers 2009; Sprod 2011), and advocates of modeling activities in mathematics classes such as G. Kaiser (2006) and R. Borromeo Ferri (2006, 2010), have proposed ways to enrich traditional classroom teaching in order to promote the active construction of knowledge by children and better conceptual understanding in science and math. Our paper presents an attempt to further this goal by philosophizing with children in the course of mathematical modeling exercises in a sixth grade math class.

Although philosophizing and mathematical modeling differ in the content of the problems they address, both approaches involve problem solving through dialogue in small groups. Because of this similarity, we felt that practice in philosophizing might have an enhancing effect on modeling activities. D. Meerwaldt (2009) has proposed several reasons why philosophizing may enhance student understanding during modeling activities. First of all, the problems selected for mathematical modeling, like the problems discussed when philosophizing with children, are complex and embedded in a narrative context. Solving them requires in-depth reflection on the part of the teacher and pupils, just as the solution of philosophical problems does. Secondly, the problem solving methods and skills employed in both approaches are similar, including cooperative learning and dialogue, analytical thinking and reasoning, critical reflection on worldviews, imagination and creative thinking. Thirdly, since both modeling and philosophizing with children are directed towards everyday problems and questions that children themselves find interesting, learning should be more meaningful. Finally, both modeling and philosophizing require that the teacher assume the somewhat novel role of a mentor rather than that of an instructor. Since it was beyond our means to test the idea of enhancement quantitatively, we designed a unit to demonstrate how philosophizing might be combined with mathematical modeling exercises in a sixth grade class, as described in this paper.

**Philosophizing with children in Germany**

In Germany, the practice of philosophizing with children was initiated in the 1920's and 1930's and further developed in the 1980's and 90's by, among others, Helmut Schreier (1993) and Ekkehard Martens (1999), both former researchers and educators at the University of Hamburg. The expression "philosophizing with children" (Philosophieren mit Kindern, PmKJ) was chosen instead of the term "philosophy for children" in order to emphasize the Socratic process involved. As Martens (2009) indicates, philosophizing in the Socratic tradition encompasses specific philosophical problems, an attitude of puzzlement and wonder, and certain types of philosophical thinking. Martens, a scholar of philosophy as well as theory and methods of teaching philosophy, maintains that philosophizing is a basic cultural technique that is just as important as reading, writing and arithmetic, and he identifies five basic forms or "methods" of philosophical thinking, which include (2009)

*Phenomenological thinking*: precisely and accurately describing something

*Hermeneutical thinking*: understanding oneself, someone else, a text or a situation through iterative interpretation; expressing one's interpretation in different ways

*Analytical thinking*: separating something into its component parts, comparing and contrasting them, classifying and naming them, clarifying the terms employed, and determining the relationships between parts

*Dialectical thinking*: viewing something from different perspectives; acknowledging, weighing and tolerating different perspectives

*Speculative thinking*: employing fantasy and imagination to explore new avenues of thought (pp. 106-109).

For some teachers, philosophizing most likely means examining a traditional philosophical topic from a field such as epistemology or ethics by employing critical, analytical thinking. Reasoning and argumentation are considered the main tools of such philosophical discourse. However, as Martens has argued persuasively, other types of thinking and their attendant cognitive skills may also be valuable for achieving in-depth understanding of a topic. Of those listed above, most philosophers would probably not question the value of phenomenology, hermeneutics and dialectics as types of thinking that might increase greater understanding of a topic, and most science and math teachers would probably agree. But the idea of *speculation* as a valuable cognitive tool is more controversial. As Martens explains (2003, pp.89-95), speculation is quite different from analytical thinking, which predominates in modern philosophy, science and math. It involves departing from objective knowledge and logical rationality and exercising imagination and divergent thinking instead. While its importance in the arts is probably unquestioned, its value in math and science is not as securely established.

Speculation is a kind of thinking and discourse that many children especially enjoy (Hausberg 2007; Höger 2007). When they are allowed to express it in philosophical discussions, their contributions may resemble "stream of consciousness" thought rather than tight argumentation, as some of the passages in the following documentation indicate. The popularity of speculation is understandable since it permits children (and adults!) to exercise what Egan (1997) refers to as mythic understanding, a manner of thinking typical of pre-scientific periods of cultural evolution and pre-literate phases of cognitive development. Egan regards this kind of understanding as a legitimate means for delving into new areas of knowledge.

The possible significance of this type of thinking has been demonstrated in various studies. Mähler (2005), for example, has examined the development of what she calls "magical" thinking, which resembles Egan's concept of mythic understanding. She finds that young children 4-6 years of age are quite capable of logical thinking but clearly enjoy magical thinking and voluntarily employ it when exploring unknown cognitive terrain. In another set of investigations philosophical discussions were used to examine children's moral attitudes toward nature (Nevers et al. 1997, 2006; Gebhard et al. 2003). The results show that when children are allowed to speculate, they often draw upon anthropomorphism as a form of imaginative, pre-conceptual thinking, which they use to better understand and moralize nature. Calvert (2000) investigated children's use of metaphors in philosophizing and discusses their significance on the basis of Cassirer's ([1953] 1994) and Langer's (1942) concepts of symbolism and presentational symbols. She has pursued the integration of creative, imaginative thinking into discussions with children in numerous practical exercises in Germany. Engels (2004) argues that fantasy and imagination are fundamental to thought experiments, which he feels are a vital aspect of creative thinking, and which Freese (1995, pp. 25ff) considers to be the ultimate method of epistemology. Finally Hausberg (2013) has demonstrated empirically that speculation is one of many skills furthered by philosophizing with children that enhance creativity. In the mathematics classroom this kind of thinking can be seen when children ponder questions like: How big is infinity? What exactly does zero mean? What would our world be like without numbers? Are numbers something people have made up? Can we really describe the world with numbers? Sometimes children pose these questions themselves in the course of instruction. Exploring such questions in math class encourages reflectiveness and intellectual ownership, as Prediger argues (2005, p. 98).

## Promoting deeper reflection during mathematical modeling

Mathematical modeling has already been recognized and put to use as a procedure for encouraging greater cognitive openness in mathematics classes (Blum & Niss 1991; Kaiser & Sriraman 2006; Borromeo Ferri 2010). It involves the use of open problems rather than closed ones – that is, problems for which more than one solution is possible. Moreover, the problems posed are complex, real-life ones that can be solved with the help of mathematical models. Problems of this kind challenge pupils to make connections between the real world and mathematics and vice versa through what is known as a "modeling cycle" (Borromeo Ferri 2006; Figure 1). To understand how a modeling exercise differs from the usual exercises presented in math classes, consider the following two examples:

A. Traditional exercise: On family day at the fair a single ride on the roller coaster costs 4 €. How often can Tim ride the roller coaster when his mother gives him 20 €?

B. Modeling exercise: A family that lives in Hamburg is planning a trip to an amusement park. They have two choices, either the "DOM" in Hamburg or the "Heidepark" in the nearby city of Soltau. Which choice is the best for the family? Present good reasons for this choice.

The first exercise requires a very simple calculation leading to an unequivocal result: $20 \div 4 = 5$. The correct answer is: Tim can ride the roller coaster five times. It doesn't matter how plausible it is that Tim would really ride the roller coaster five times, nor whether he would really choose to spend 20€ this way. The calculation is correct even if the exercise is meaningless. The modeling exercise, on the other hand, is embedded in a larger narrative context. Completing the modeling exercise requires that the children understand the greater context of the problem, elaborate it, and establish their own parameters for finding a solution. They have to decide how many members the family has, how much money is available for the trip, the day on which the trip is to take place, how the family plans to get to the park and the costs that ensue, whether they want to have lunch at the park or not and the expenses involved, and what rides they want to take. An exercise of this kind is open, complex, realistic, authentic and solvable with the help of mathematics (Maaß 2007, p. 12). Furthermore, solving it and comparing different solutions requires dialogue among the pupils.

The following modeling task was used in the study described in this paper. We would like to illustrate how the modeling cycle works on the basis of this task:

One winter day when the teachers and children arrive at their school, they discover that someone has broken into it. There are footprints in the snow that have probably been made by the thief. In order to investigate the crime the police want to rope off the school so that no one else can enter it. 1.) How much rope do the policemen need to close off the school? 2.) Judging from the footprint left behind, how tall was the thief?

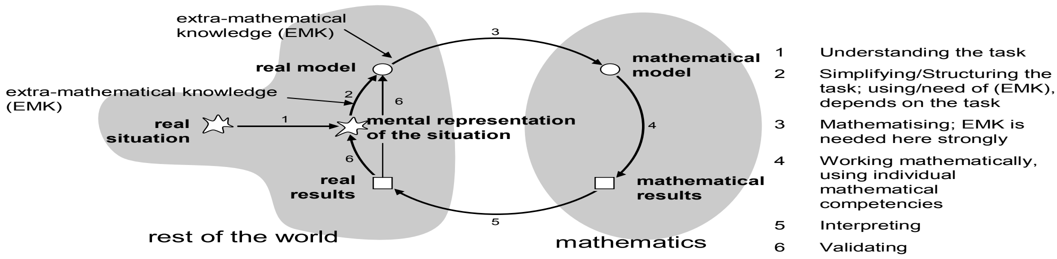


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Figure 1. The modeling cycle according to Borromeo Ferri (2010)

The real situation in the cycle, depicted in Figure 1, can be presented to the children in different ways – in written form, for example, or as a picture. In the case in point the situation was presented as a story. The problems posed by this situation are to determine exactly how much rope the policemen need to close off the school and how tall the thief is. The second step in the cycle is the so-called mental representation of the situation. This occurs at an unconscious level, but as we know from empirical research, in order to understand the problem it is necessary for the children to visualize the given situation. Specifically this means that the children must imagine their school building and the rope; perhaps they may also imagine the janitor, the broken door and the policemen. This provides an opportunity for many different kinds of associations.

The pupils then simplify the problem and make some hypothetical assumptions. In the present case they must decide which part of the school building should be closed off – the whole building, for example, or only part of it. To do this, the children have to deal with the floor plan of the building, and they have to acquire or apply knowledge about scales and measurement. The next step is to create a mathematical model based on these assumptions. This can be done in several different ways. One possibility is to directly measure the lengths of rope determined as necessary on the basis of the floor plan, add them up, and then convert this value to meters or some other standard unit of measurement. Mathematical competencies required to perform this step include addition, multiplication, dealing with several different units of measurement and conversion. Depending on the assumptions the children make, the mathematical results can range from 8m, if the main and the side entrances are closed off, to 350m, if the entire school is enclosed. The pupils have to interpret their results and then decide whether or not their results are realistic. This process of weighing and evaluating results is called validation. If the children come to the conclusion that their results are not realistic, they have to reassess their modeling process (Blum and Borromeo Ferri, 2009). In the present study the first steps of this procedure were performed in small groups, while validation was conducted in a plenary discussion. Both parts require dialogue.

For the second part of the modeling exercise concerning the size of the thief, the children were given an outline of a footprint on a piece of paper. They were asked to imagine the real footprint and the thief who produced it. Perhaps they also might imagine the kind of shoe that would produce such a footprint. Then they must decide what measurements they need. To solve this problem they have to estimate the relationship between shoe size and body size and apply this to the footprint. To do this they might measure this relationship with one member of their group or determine the average value of measurements derived from several different people. At any rate, a mathematical relationship between shoe size and body size is determined and applied to the footprint they have been given, which is 32 cm long. Using algebra they can then estimate the size of the thief. Assuming that the relationship between shoe size and body size is 1:6, the mathematical result would be 32 x 6 = 192 cm. In reality the size of a person with a footprint corresponding to European shoe size 46, which is 32 cm long, could be between 180 cm and 200 cm.

**Design of the investigation**

In order to examine the possibilities of combining modeling and philosophizing, a sixth grade class at an upper level school ("Gymnasium") in Hamburg was chosen that included 10 boys and 18 girls. The class had had no experience with modeling, but it had been philosophizing since the beginning of the fifth grade; some of the children had even begun in primary school. Thus the class was well on its way to becoming a community of inquiry even before the study was performed. Since successful philosophizing requires practice, it was thought that the prospects for an observable influence would be more promising with this class than with one that had had no such experience. A unit was developed that covered 8 hours of class time and included the modeling task described above. Before being exposed to the modeling task, the class had already had experience in calculating the area of surfaces.

At the beginning of the unit the class was divided into two groups. One was given the opportunity of philosophizing – in three different group discussions – about different topics related to the modeling task, while the other half worked individually drawing and making posters without communicating. Then both groups were given the same modeling exercise. We hypothesized that if philosophizing has a very direct and immediately observable effect, the two groups might differ in their modeling performance. As described above, solutions to the problem were developed in small groups and presented in the form of posters. Then the various different solutions were evaluated in a plenary session referred to as validation in the modeling concept. During different phases of the investigation the groups were observed through video documentation and field notes. Both the videos and posters were analyzed as part of the investigation. In addition, the pupils were asked to complete a questionnaire at the end of the unit. The following questions served as guidelines for the analysis of the data we gathered: *What effect does philosophizing have on the modeling process? Are the students in the test group better able to understand the problem? Do they communicate differently than the control group while working on the modeling exercise? Does the test group show any differences in validating its results? Does the entire class exhibit skills in modeling that could be attributed to their past experience in philosophizing?* Obviously a limited case study of this kind does not permit any scientifically valid conclusions to be drawn. However, as a heuristic procedure it might reveal interesting tendencies and raise valuable questions for further study.

Regarding the content of the problems addressed while philosophizing in a math class, an obvious choice would have been problems in the philosophy of mathematics such as the handful listed above (p. 5). However, to establish a more direct connection between philosophizing and mathematical modeling, we chose topics more directly related to the specific modeling task used that might enhance understanding of this task, its narrative context, and the nature of the process of mathematical modeling. In the test group three different topics and three different types of philosophizing were employed over the course of three different discussions. The first discussion dealt with a matter of ethics and had nothing to do with mathematics per se. It is based on a well-known story derived from Lawrence Kohlberg's studies of moral development – the story of Heinz, who contemplates breaking into a pharmacy to steal medicine for his dying wife. Since both the modeling task and the story involve theft, the Heinz Dilemma was chosen as a warm-up exercise to encourage the children to consider possible motives and explanations for theft in the modeling task. The second discussion had to do with clarifying concepts, although in this case the concepts are not ones typically discussed in philosophy classes. Instead, the terms "mathematics" and "reality" were explored conceptually. Since modeling involves relationships between mathematics and reality (see Figure 1), it seemed reasonable to address this relationship before the concept of modeling and the first modeling exercise were presented. The third discussion journeyed deep into the realm of speculation and involved imagining how a person might succeed in leaving no footprints behind, and what the consequences would be. This last discussion was carried out before the children were asked to estimate how tall the thief was on the basis of his footprint. All three discussions were moderated by a facilitator as group philosophical discussions. The math modeling unit ended with a plenary discussion, in which the children compared and weighed the different mathematical solutions they had arrived at. Although this discussion was not moderated, it closely resembled a philosophical discussion, since it required argumentation and persuasion.

**Results from the three philosophical discussions**

1) The first student response to the Heinz Dilemma was not one of ethical query, nor was it comparable to a strongly focused discussion about saving life vs. property rights such as one might find in a philosophy class. Instead, the children began thinking about ways to raise a large amount of money in order to prevent the protagonist, Heinz, from having to steal medicine. In addition they talked about the price of drugs and the financial situation of doctors and hospitals. This suggests that the everyday thoughts of children in response to the story may be quite distant from the ethical dilemma that the story is intended to induce. However, later on the discussion did take a more traditional turn, as the following excerpt illustrates:

M: *We have to take a radical view of the situation. He [Heinz] has tried everything.... Now all he has is the decision to steal or not to steal.*

...

S1: *Well, I wouldn't steal anything, because it's like, like you do something that's against the law, and if you rob somebody, like, I'd have a guilty conscience, because then the pharmacist would be hurt too, because the medicine is gone, so to speak. Besides, the pharmacist would know right away who did it since he knows how badly he needs it, and everything would be useless anyway.*

M: *So you wouldn't break into the pharmacy.*

S2: *I would, I mean, I don't know exactly. I'd break in, because then he'd have helped his wife, but then he'd land in jail, and he'd be punished, but his wife would be alive.*

S3: *Well, it all depends. If his wife says "Oh, I don't want to live anymore," then I wouldn't do it. Or else I would, another possibility would be to go to the government and describe the situation, because even if the government can't help him with the theft, it just can't be that if there's a medicine that could keep her from dying, then the government should really take over, because it can't just let a woman die. So he could really go to the government, and if that doesn't work, then ...*

M: *So you're saying it's not just Heinz's problem, that it's really a problem of the entire society?*

*S3: Yes, because then you'd read in the newspaper "Government lets woman die although medicine is available!" That wouldn't be so good either.*

*...*

*S1: Well, I also think he should do it [break into the pharmacy], because then he'd feel better if he'd done something good, because he'd have saved a life, so to speak, because it's not so bad if the pharmacist loses some medicine than if a human being has to die, that such a stingy person lets her die.*

The children continued to talk about what it would be like to have a guilty conscience, the pros and cons of assisted suicide, and the extent to which we are able make an autonomous decision about life and death. The moderator made sure that the children honored the rules for a philosophical discussion that they themselves had established, and that they built upon each others' contributions. She also summarized the discussion at various points and insisted that the children give reasons for their views.

2) A philosophical discussion about the concepts "mathematics" and "reality" was included since we felt that it might have some value in helping students understand what modeling entails. First of all, the children were invited to individually prepare a conceptual map for each of the terms. Then a group discussion was initiated by asking each pupil to express briefly a thought he or she had regarding the terms. The associations the children mentioned included: fun, numbers, language, boring, tests, solutions, one of the co-investigators, teachers, teachers' pets, man-made, calculations, addition, lots of homework, death. Most of these associations appear to reflect the children's view of mathematics as a school subject. Then the moderator asked the children to explain what math is on the basis of what had been said and began a moderated discussion in the traditional manner of Philosophy for Children. Excerpts from this discussion are presented below.

*S1: Well, it consists of lots of numbers, a whole lot of homework and tests.*

*S2: Mmm, you need it all your life for, I don't know, for example, for shopping.*

*S3: It's fun to be challenged. But if you're not challenged, it's boring.*

*S4: I would say it's something like a language, because, you know, there are many different languages and most people can do math. That's why for me math is a language.*

*...*

*S6: Well, I think it's something man-made, to somehow get along better in life. But when you think about it, if math hadn't been made, then we wouldn't have needed it. Because then it would be something like an apple that doesn't even exist.*

*M: Oh, that's an interesting question, whether or not we would need it if we didn't have it. You mean that then we wouldn't have any numbers and nothing that has to do with numbers?*

*S6: Well, then, in that case an apple would be, sure, then somehow, somehow, I don't know, but I think in that case the thing wouldn't be called a euro. It would be called something like a piece of gold.*

*...*

*S7: Mmm, I'm not so sure. Sooner or later people would have figured it out anyway, because they would, well, just think if they had ten pots and would say "I have pots." But then nobody knows how many and so you have to say it somehow. Then maybe they would use their fingers to show how many they have and sooner or later there are numbers.*

*M: You mean that numbers would come to be in any case?*

*S7: Yes, and I wanted to disagree with S1, because she said that mathematics consists only of numbers, but it consists of much more than numbers. There's geometry too.*

*S4: And letters.*

*S8: And I wanted to say, I mean I think you could also do it this way, if you had ten pots, you could say pots, pots, pots, pots, pots, pots, pots, pots, pots, pots.*

*S3: You mean pot, pot, pot.*

*S8: Yeah, exactly, that you say pot ten times, and when you then say pots, that could mean 10 pots. And when you say pots, pots, pots, then that's thirty pots.*

*S7: But then, actually, you're back to numbers, because then, in principle, you have numbers, because for the number ten you have a symbol, because when they say pots it means ten pots, so to speak, and that is like our ten, so to speak. Then that's a number too.*

The children's comments indicate that they see mathematics as something useful for shopping or "to get along better in life." They also regard it as something personally meaningful since it can be fun. They are able to conceive of math as something with a structure like a language that can be learned. And they explore the significance of math by proposing a thought experiment, which allows them to go beyond the commonly held view of mathematics as a school subject and discuss questions like: What would it be like if we had no numbers? What are they good for?

3) The third discussion – about what kind of human being would fail to produce a footprint – brought the children back onto the track of the modeling story about breaking and entering in their school. The first idea one child expressed was that it might be a person in the form of a shadow. This led to other philosophical questions: Is a shadow a person? What makes a living thing a living thing? Another child suggested that a person who produces no footprints must be God, because there is no evidence for God, no "traces." And yet another expressed the following idea:

*Well, I think a person without footprints doesn't exist anyway, a man, a human being who doesn't leave any traces behind, but if he did exist, then there would be no real justice anywhere on earth, because then he could break into a place without leaving any traces behind.*

This discussion also gave the children the opportunity for creative speculation by imagining various different ways that one might avoid footprints:

*That must have been someone who floats on air.*

*You'd have to have a kind of tank on your back that lets you breathe in and out.*

*If you had a kind of jumpsuit made out of some kind of material that doesn't allow anything to pass through it, then you could walk or touch something without leaving any traces behind.*

*He could have a board that is weightless and floats on air and then some kind of shoes, there must be something like that, that are magnetic or something and don't leave any footprints behind.*

While the value of such a discussion for learning math may not be immediately evident, its significance lies in opening children's minds to imaginative and creative modes of thinking. Providing an opportunity for divergent thinking is also in keeping with the "flexible attention policy" exercised by the highly innovative Minnesota 3M company to encourage creative thinking, as reported by Lehrer (2012, p. 29). According to this policy 3M employees are allowed to spend 15% of their workday pursuing speculative new ideas rather than focusing on the project to which they are assigned, a period which the employees call their "bootlegging hour". Perhaps speculating in the course of philosophizing in different subject areas could have a similar effect on children.

*Results of combining mathematical modeling and philosophizing*

Analysis of the videos and protocols used to document the unit shows no striking difference in communication among the pupils in the test group, who philosophized prior to beginning the modeling exercises, compared to the control group during the modeling exercises. Communication in the test group was not any more respectful, open, or elaborate than that of the control group. Instead, other factors such as individual personalities, group dynamics and gender seemed to be more significant for the kind of discussion that ensued. A study with a control group without any experience in philosophizing might provide further insights into the effects of philosophizing on communication styles. However, it is extremely difficult to find a valid control group that is similar to the test group in all respects except for experience in philosophizing. It should also be noted that the discussions among the pupils during the problem solving and validating phases were not moderated by an adult. This is very different from the moderated discussions commonly used when philosophizing with children. In a subsequent study involving the solution of an open problem in a biology class (Hausberg 2013), the validating phase was conducted in the manner of a moderated philosophical discussion. In this case, the validating phase resembled a philosophical discussion much more distinctly than the discussions in small groups prior to the plenary discussion, which were not moderated. In the present study, the pupils in the test group did seem to be more engaged than the control group when the various different groups first presented their solutions. They seemed to question the solutions of other groups more actively. They were also more critical than the control group during the validating phase. It is not, of course, possible to attribute these differences with absolute certainty to the period of philosophizing that the test group experienced, but it seems likely. In general, the entire class exhibited a high level of reflective competence and was unusually active in validating compared to other classes observed informally during modeling exercises. Since the class observed here was well trained in philosophizing, it is quite possible that this resulted in the greater ease in validating observed.

The results of the questionnaire indicate that the class in general exhibited a high level of tolerance towards tasks with several different valid solutions. Eighty three percent of the pupils reported that they enjoy exercises with different solutions. This is contrary to claims commonly made by teachers that pupils are easily frustrated if they are not told by an authority which answer is the "right" one. In a final feedback session the pupils in the test group agreed that they enjoyed the combination of philosophizing and mathematics. They were more inclined than the control group to see a connection between philosophy and mathematics. Moreover, compared to the control group they felt that they had had a greater opportunity to apply their everyday knowledge to the modeling process and found the modeling procedure "easy."

## Concluding Remarks

Previous research has revealed various problems associated with using the modeling process in the classroom. First of all, modeling requires more independent thinking on the part of pupils than is usually expected in mathematics classes. Moreover, they must be able to apply many more skills than simply calculating a particular result. They have to imagine a situation, propose a solution, discuss it with others, reason with them and weigh their results compared to others. For many pupils it is highly irritating to discover that several different solutions to one and the same problem are possible, and some find this difficult to bear. Research on the modeling procedure has shown that certain beliefs about mathematics have a strong influence on the success of the solution process, and that voluntary reflection and validation are often skirted (Borromeo Ferri 2010). Children still seem to search for the "right" answer and turn to the teacher for validation.

The investigation reported here suggests that experience in philosophizing in math class as well as outside of it might serve to mitigate some of these problems. Regular exercise in pondering und discussing open questions in the course of philosophizing appears to promote an attitude amenable to dealing with open problems in math and other subject areas as well. Children learn to accept the fact that there may not be a single "right" answer to a particular problem. They learn to think independently and to question their own thoughts as well as those of others. Furthermore, they learn to examine and question the nature of certain bodies of knowledge such as mathematics or history or science, thus assuming a critical, meta-cognitive position to such knowledge. In order to optimize its effects, we believe that practice in philosophizing should begin at an early level of schooling and be made a regular part of the school curriculum. In addition, it is important that philosophical discussions be clearly earmarked as such to avoid confusion with other learning processes. Philosophizing can then provide space for children to express their thoughts freely and relieve them from the pressure to produce a "right" answer so common in many other learning situations, even though producing a "right" answer may well be justified in these contexts.

If philosophizing is to be established as a basic educational practice that can be exercised in subject areas like math or science, it would be important to improve the philosophical background of teachers in these areas, as W. Turgeon (2011/12) has also suggested for teachers active in P4C in the USA. An obvious solution would be to make training in philosophy and philosophizing mandatory for primary and middle school teachers. However, this training would have to be tailored to the specific needs of teachers. The problem cannot be solved by simply requiring all teachers to obtain a major or minor in philosophy since the aims and methods of these courses of study at most universities are usually far removed from the interests of future grade school teachers. There can be a great difference between "studying philosophy" and "philosophizing," and the skills of the latter are not necessarily learned in the former. As such, training teachers in facilitating children's philosophical conversations represents a new sub-field within philosophy, with strong implications for the future of childhood education across all the disciplines.

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