Beyond Epistemology and Axiology: Locating an Emerging Philosophy of Mathematics Education

Nataly Chesky

Abstract

This paper explores the need to move beyond epistemological and axiological discussions in philosophy of mathematics education by reframing the inquiry to include an ontological perspective. The main goal of this work is to envision a new relationship between philosophical discourse and mathematics education, one that takes into account ontological assumptions in mathematics and relates it to axiological objectives and epistemological claims. I begin with a description of the dominant view of mathematics education as depicted in U.S. policy reform discourses and contrast it with critical views of philosophy of mathematics education. I show that the deficiency of both perspectives is their lack of attention to the ontological assumptions inherent in mathematics education. Lastly, I provide a preliminary sketch of a new type of philosophy of mathematics education, which brings ontological inquiry to the forefront, and I speculate on how this emerging philosophy of mathematics education might impact the field.

Introduction

Philosophy of mathematics education can be classified dichotomously based on the educational objectives that prescribe the goals mathematics education should strive towards, and the pedagogical theories that suggest effective teaching and learning strategies. We can call the former an axiological objective, since it places a normative value on educational goals, and the latter an epistemological claim, since theories on teaching and learning are always grounded on claims of how knowledge acquisition works. Both the axiological and the epistemological discourses are present in the dominant views of mathematics education as depicted in national policy discourses in the U.S. as well as in the more critical views of mathematics education that are present in international scholarly venues. On the one hand, U.S. policy views of mathematics education are predominately interested in fostering strong cognitive competencies in mathematical problem solving. Those that hold these views, while they pay homage to constructivist pedagogical approaches, tend to believe that mathematics education ought to emphasize content knowledge and expertise in computation for utilitarian ends, such as job skills, ensuring national economic competitiveness, and maintaining math literacy for citizenship and consumerist purposes such as filing taxes and understanding interest rates on credit cards (e.g. Spillane, 2000; Steen, 1997). On the other hand, critical views in philosophy of mathematics education emphasize both constructivist-learning pedagogies and the societal/cultural origins of knowledge. Those who adhere to these views believe mathematics education ought to be used for raising critical consciousness in order to utilize the power of mathematical discourse to uncover social inequalities prevalent in modern day Western society, which, it is believed, will inform the praxis of mathematics students and enable them to become change agents in their communities and in their personal lives (e.g. Ernest, 1988; Frankenstein, 1983; Gutstein, 2006; Skovsmose, 1994). I will argue in this paper that what is explicitly missing in both of these philosophical views of mathematics education is an ontological perspective. Not only will taking ontology seriously help ameliorate the respective goals of both of these philosophies, but it will also add a new integral component to each that has the potential to help shape the future direction of mathematics education in the U.S.

By ontological, I am referring to the way in which mathematics and the entities/objects/processes that are utilized in its practice explain a model of the world as we have come to understand it. Ontologically, we can say that an absolutist vision would be very different from a fallibilistic one in that the former would posit essential, unchangeable quantifiable objects, dimensions and relationships of the being of the world, and the latter would argue that the world is either unknowable or continuing changing and therefore immeasurable in the traditional sense. Unlike epistemology and axiology, ontology has not been represented well in the discourse of philosophy of mathematics education. This, as I explain later in this paper, is unfortunate, since the link between epistemic claims about how we gain mathematical knowledge is directly related to where we believe such knowledge is located and what exactly such knowledge consists of. Whether one posits a purely semiotic view of mathematics, as nominalist do, or a purely mental construct game of finite symbols, as intuitionists believe, ontological assumptions are implicit. After all, it is ontological assumptions that underlie all pedagogical theories and axiological objectives, and therefore ontology has a fundamental role to play in mathematics education discourse.

This paper is separated into three parts. First, I provide greater detail on the two perspectives of philosophy of mathematics education that I mentioned above, and I explain how they might be improved by incorporating an ontological perspective. Second, I delve deeper into my claim that an ontological perspective is needed in mathematics education. Lastly, I theorize in what ways a philosophy of mathematics education in which the ontological dimension is recognized and explored might play a role in mathematics education in the future.

The Dominant Perspective of Mathematics Education

The current widespread public discourse on mathematics education dates back to at least the dissemination of *A Nation at Risk* in 1983, if not to the 1957 Russian Sputnik Launch. This was a crucial historical moment for United States policy makers and, in turn, the public at large--the moment in which the former announced that we should be alarmed at our lack of mathematical abilities in relation to other nations.[1] Soon after the hysteria generated by the "report" dissipated, progressive educational approaches moved in, eager to reform traditional mathematics education. "New Math" was the term given those pedagogical alternatives that sought to provide the mathematics learner with both a holistic and an abstract understanding of mathematics. This reform effort failed for conflicting reasons, and its detractors proclaimed it to be elitist, leaving a generation of mathematics students disinterested at best and antagonistic at worst toward the subject of mathematics (Klein, 2003). After the perceived failure of "New Math," the pendulum swung in the opposite direction with "The Back to Basics Movement," which returned to a more traditional model that emphasized the learning of algorithms and procedural knowledge rather than conceptual or theoretical understanding.

Over the last several decades, mathematics education has increasingly taken center stage in educational policy discourse. Particularly, after the 2009 PISA (Program for International Student Assessment) results were released, which depicted United States students as mediocre in mathematics skills compared to their international counterparts, the national reform movement in mathematics intensified. Most recently, the "STEM" (Science, Technology, Engineering, and Mathematics) policy reform initiative has taken the national stage for exemplary mathematics education reform policies. STEM is not only a mathematics education reform package; it is also an interdisciplinary education policy combining science, technology, and engineering with mathematics. This is significant, since past policy reforms, driven by the need to maintain global competitiveness, concentrated on mathematics, foreign language, and science education. The differences may lie in the means believed to be needed to achieve this strengthening, as well as the deeply entrenched

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assumptions about what mathematics is and how it can be utilized.

Berry, Ellis, & Mark (2005) argue that the "reforms" in mathematics education were merely revisions since they do not qualify as elements of a true paradigm shift: they do not, that is, offer a different conception of knowledge, nor do they provide an essentially different pedagogical approach that would benefit the historically marginalized population of students that often times do not gain access to higher level mathematics knowledge. In other words, the changing axiological claims in policy reform discourses have not been significantly different since they are founded on similar, if not identical, epistemological and ontological ideas about mathematics. Moreover, while reforms have emphasized different values mathematics education ought to serve such as democratic, cognitive, or utilitarian, their stance on where mathematical knowledge comes from and how best to teach it has not drastically changed. The discipline is still conceptualized according to an absolutist vision of mathematical entities--i.e. that numbers and functions exist regardless of human intervention or invention. The question for the dominant mathematics education discourses is then, how can such reforms achieve their own axiological objectives if they do not alter the way mathematics itself is taught, learned, and thought about? Of course, this question does not call into doubt the rightness of the axiological claims being made, which is the primary target of critical theorists of mathematics education. However, as I explain in the next section, the critical perspective on mathematics education also remains reductive insofar as it does not situate itself within the complexity of the discourse of philosophy of mathematics as it relates to educational issues prevalent in the current U.S. political and educational climate.

The Critical Perspective in Philosophy of Mathematics Education

Many scholars oriented to critical theory (e.g. Apple, 2005; Atweh, 2007; Giroux, 2005) have vehemently argued against recent policy practices such as *No Child Left Behind* and President Obama's *Race to the Top* initiative for being ill-conceived, nondemocratic, and detrimental to the quality of our public schools. These scholars insist that mathematics education has served the elites, and subjected the working class to alienation and incomprehension. In order to address this social injustice, they have formulated two primary objectives. First, mathematics can be used to uncover such class inequalities by means of pedagogical techniques that foster inquiry into the social realities of students' lived experience. Second, mathematics itself should be exposed as a socially and culturally derived discipline that is not strictly tied to the Western epistemological lineage.

One type of critical philosophy of mathematics is critical mathematics education (e.g. Gutstein, 2003; Skovsmose, 1994), which was inspired by the work of Paulo Freire, who proclaimed that revolutionary leaders must also be educators. Freire's epistemology is antithetical to the Western positivist paradigm since it views mathematics knowledge and education as never neutral; rather than a set of value-free objective truths, mathematics is seen as creating dichotomous power relations among different groups of people and then legitimizing these dichotomies to serve the needs of a ruling class. Freire saw how "massified" consciousness is more prevalent in technological societies such as ours and is a major factor in determining the inability of subjugated people to actively engage in their own revolutionary agendas. Therefore, developing critical mathematics pedagogies becomes increasingly urgent as societies become more technologically saturated.

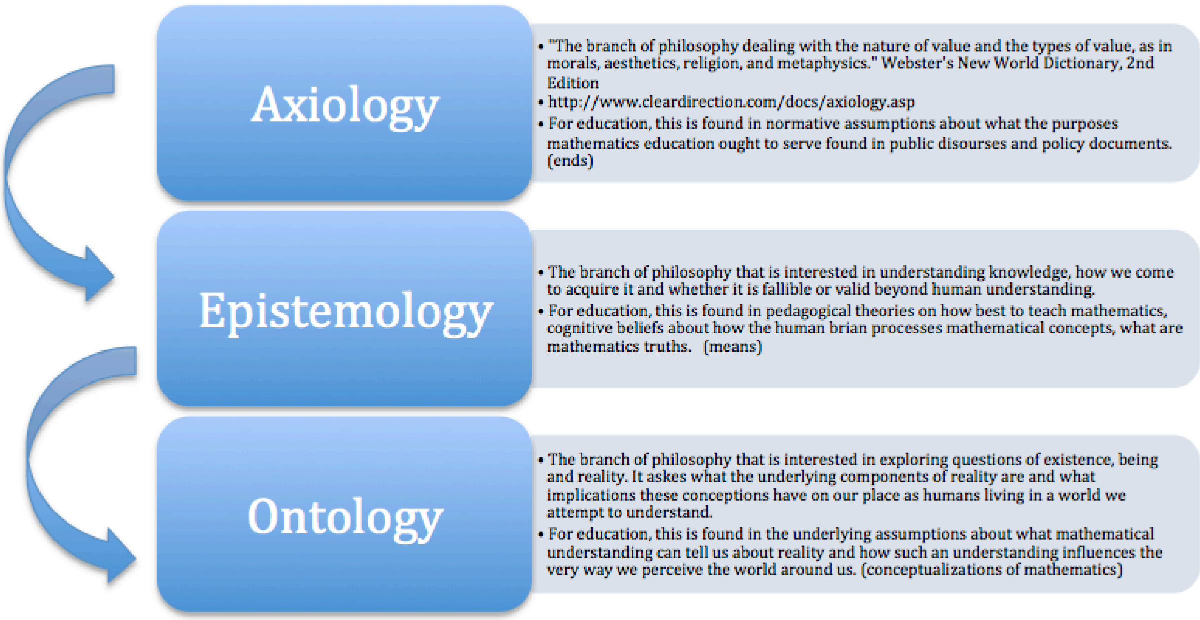
Ethnomathematics (e.g. Frankenstein, 1983, D'Ambrosio, 2001), another critical perspective in philosophy of mathematics education, attempts to uncover the cultural foundation of mathematics by stressing that math knowledge is always generated in a historical context. Ethnomathematics understands itself as a critical theory of pedagogy that attempts to resist hegemonic Euro-Western ideology in order to reestablish epistemological alternatives that are found in indigenous cultures. Ethnomathematics certainly has much to offer, in that it broadens our cultural awareness of indigenous cultures, critiques Western positivist claims on mathematics knowledge, and puts into ethical question how mathematics has historically marginalized certain groups of people. The weakness of this educational alternative is that it has little epistemological support, and does not take into account inherent ontological assumptions about the nature of mathematics and how these influence cultural views on the field. Katz (1999) argues that there is an epistemological incoherence in ethnomathematics, since there is historical proof that mathematical "discoveries" have arisen in separate locations--for example, the Chinese and the Greeks independently figured out the Pythagorean theorem and Pascal's triangle. Furthermore, ethnomathematics does not take into account the political and historical events that have led to the marginalization of certain forms of knowledge. For instance, teaching urban U.S. students about African villages does little to give them an understanding of how and why such villages have been colonized and continue to be places of intense human hardship. More to the point, teaching villagers in Ethiopia about their own culture's contributions to the discipline of mathematics lends little real support in their political and personal struggles to survive in a globally connected world that is dominated and controlled by a use of mathematics that tends to enforce rather than alleviate their marginalization. After all, like it or not, mathematics has a strong hold on Western consciousness, and providing alternative examples is not enough to break that hold.

Not only is mathematics unanimously valued as a field of knowledge in our modern technological western society, it is also the cornerstone of the rationale abstract paradigm that many have argued defines our society's current state (Brubaker, 2008). Hence, philosophers of mathematics education should ask themselves, in what ways does mathematics influence the society in which we live and how might they best counter or make more transparent such power? Furthermore, we ought to seek out the fundamental causes of mathematical power over us. As Neil Postman wrote: "We must be aware not only of how to use mathematics, but also how mathematics uses us" (Warnick & Stemhagen, 2007, p. 304). This form of power is prevalent, not only in the way in which our capitalistic system works and our technological devices operate, but also in our rationales for educational assessment, teacher evaluation, and federal accountability legislation. But I want to suggest that by turning a critical eye on mathematics itself rather than just on its oppressive uses, math educators can assist us in changing the way in which we normatively believe the world operates, and can suggest to us what type of agency critical citizens might have in such a world. As it is, although critical mathematics pedagogies strive to empower students by enabling them to gain the tools needed to "read the world," and thus to transform it (Atweh, p. 7), we have no empirical proof that there is a causal relationship between becoming aware of social inequalities and becoming politically active in order to bring about change. One possible cause of this disconnect may be a failure to take into account, not just the epistemological but the *ontological* assumptions of mathematics.

For example, a positivistic/empirical stance corresponds, as I have already suggested, to an ontological view that there are indeed certain entities in the world that can be described mathematically, which leads to the notion of a universal concept of number as being outside human social construction. Inversely, a formalist or nominalist approach to mathematics implies a view that mathematical phenomena do not exist apart from their historical social context. Positing one or the other of these two extremes in the context of democratic activism can lead to drastically different results. On the one hand, a positivist stance uses numbers without questioning their own relative relationship to one another. On the other hand, an educator with a more formalist ontological perspective highlights how quantitative data is used statistically with few questions about the efficacy of its truth claims. For example, an educator with the first view might show us numbers that "prove" that more African Americans get sentenced to prison than their racial counterparts--but what does this statistic mean, and what other statistical information is needed to make more sense of it? Indeed, the current standards movement in curriculum and accountability for teachers and teacher preparation programs is founded solely on a positivist ontological stance toward numbers that does not question the validity of numerical data, nor does it ask in what ways such data is variable and related to other social data that cannot be quantified. As such, these ontological assumptions do, in fact, affect our lives and our ways of perceiving the world. By making these assumptions explicit, perhaps a more complex view of mathematics might arise that can add to both the positivist and the formalist perspectives on mathematics education.

The Missing Ontological Perspective

Please consider the diagram below to explain the relationship between axiology, epistemology, and ontology in mathematics education.



This diagram illustrates that axiological objectives in mathematics education presuppose epistemological claims as to how mathematics can be learned and thereby how it should be taught. As Thom (1973) suggests, "All mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics" (p. 204). Particularly in the late 19th and early 20th centuries, debates about the nature of numbers and the foundations of mathematics were widespread. Unfortunately, very little work has been done in mapping these debates in philosophy of mathematics education. The following section attempts to give a preliminary sketch of how we might do so.

As I have already suggested, there are traditionally two ways of conceptualizing the field of mathematics--absolutism and fallibilism. The former believes that mathematics has a direct link to empirical or rational truths outside the human subject, while the latter posits that all mathematical knowledge is based on cultural, social, and political forces, which are inherently flawed, evolving, and biased. Absolutist theory includes realism and some forms of formalism and intuitionism. Fallibilist theory includes nominalism and constructivism (Ernest 2004). This simplistic dichotomy leaves much to be desired. For example, whether or not we posit an ontological status for mathematical truths or not, it is unclear how pedagogical practices might to be affected by the difference. Indeed, neither seems very satisfactory given the complexity of the debates in current philosophy of mathematics discourses. Each camp argues for their own view by critiquing the others or ignoring them all together. Fallibilist theory ignores the paradigmatic paper published in 1960 by Eugene Wigner, titled "The Unreasonable Usefulness of Mathematics," which argues that there is an uncanny correlation between abstract mathematical formulas and the way they accurately describe natural phenomenon as experienced and understood by scientists (Burbaker, 2008). By ignoring the empirical uses that mathematical abstraction continues to play in science, formalist accounts of mathematics lose credibility. On the other hand, by ignoring the existence of paradigm shifts that have led, for example, to the unanimous acceptance of the fallibility of Euclidean geometry, absolutist accounts of mathematics appear stubbornly rigid and illogical.

Taking ontological assumptions into account when theorizing about ways to teach mathematics for optimal learning would not only introduce a healthy multiplicity into the field, but it would also help meet the dominant aims of mathematics education. Take, for instance, a study done by Chiou & Anderson (2009) with undergraduate physics students, designed to assess their formative understanding of heat conduction based on mental model theory and an ontology-process analysis. The researchers found that "ontological aspects of the students' mental representations including a student's ontological beliefs, a definition, a presupposition about the ontological nature of things, i.e. the representational entities or elements that comprise an interpretation of phenomena" did, in fact, influence how students successfully acquire high level mathematical ideas (p 828) Another researcher found that children's mental models are, in fact, built upon the constraints of their own ontological and epistemological beliefs (Brewer, 1994). Brewer argues that studying children's ontological beliefs and conceptual understanding will aid in facilitating their learning. By showing learners the structural and relational nature of mathematical concepts and entities, and laying bare the misunderstandings in their own assumptions about numbers, learners might feel more confident in their own process of learning mathematics. For young learners, this could be achieved by explaining the composite nature of numbers through an exploration of our base-ten system, and the use of a 100 chart to show the relationships between numbers. For middle school students, lessons on infinite numbers and transfinite numbers might greatly increase their understanding of mathematics early on so they do not have to wait for higher level high school mathematics, by which time most of them would either already have developed a dislike for the subject or have been excluded due to their grades.

Exploring ontological presuppositions are also very relevant for the critical perspective of mathematics education, which rests upon constructivist pedagogies. Current constructivist pedagogies do succeed in changing the dynamics of a classroom and moving the power relations away from an authoritative teacher. This may help critical pedagogy's axiological goal of raising the critical consciousness of math students, but without questioning the way in which Western society has conceptualized mathematics and thereby utilized it to serve exploitative and arguably unjust systems and practices, critical consciousness cannot be reached. This is because mathematics itself shapes our perception of reality. Fisher (2006) writes, "Math is a means which we can use, and simultaneously it is a system to which we are subject" (p. 318). Therefore, to gain the critical consciousness Freire was advocating, we must become aware of the ways in which our society uses mathematics, and of how such usage affects our very understanding of our world and ourselves. By understanding the way in which our conception of mathematics influences the way we categorize and make sense and meaning of our world, we will be in a better position to critique and finally to change it.

When we speak about democracy and access to knowledge as a social justice objective for mathematics education, what exactly are we assuming about the nature of mathematics itself and the way it ought to be used in our society? This is not only an epistemological and axiological question, but an ontological one as well. Mathematics is so integral to our technologized world that is it foundational to the way we perceive the world. Therefore, any philosophy of mathematics education must at the very least implicitly proclaim its ontological assumptions, argue for their value, and argue for how they are consistent with best practices.

An Emerging Philosophy of Mathematics Education

Ernest (2004) suggested that we ought to view philosophy of mathematics education not as a single position, but as "an area of investigation" (p. 1). Traditionally, philosophical research in education has focused on the practices of teaching and learning of mathematics, exploring what cognitive theories are best for mathematics learning objectives and what types of classroom organization best facilitate learning of mathematics. If we take Ernest's suggestion seriously, a new type of philosophy of mathematics education emerges--one that makes use of philosophical discourse as a reflective meta-language, which we can use to study the normative assumptions that inform the way we conceptualize mathematics, and how we put that conceptualization into practice in the classroom. Hence, this kind of philosophy of mathematics education would seek to understand how the discourses surrounding mathematics education could be understood in relation to entrenched societal values and beliefs and covert assumptions about how the world ought to operate and what functions humans play in such a world. A new research agenda emerges from this as well—one that takes into account the complexity of the discourse and is equipped to analyze it through a philosophical lens. By examining the ontological and epistemological assumptions embedded in policy discourse about mathematics education, we can better assess if they are coherent with our axiological aims.

For this new attention to our implicit philosophical assumptions to be of use to classroom teachers, who are daily striving to help their students learn actual mathematics—and driven as they are by seemingly entrenched policy mandates—we must begin with the basic existential premise that mathematics, both as a discourse and as a set of concrete practices, frames our perceptions about the world. These perceptions then affect how we view political ideologies, educational practices, attitudes and beliefs about assessment, theories about how our minds learn, the status of economic pursuits as means to happiness, and relationships within and between our communities, both locally and globally. The emerging philosophy of mathematics, as it is sketched here, allows us to see the significance of mathematics as a societal knowledge system, and thereby grasp its crucial importance as an educational subject in a more complex way than either the critical theorist or the positivist policy makers can, given their inattention to the implicit beliefs that drive their practice.

Conclusion

Privileged voices in the conversation about mathematics education tend to claim, either explicitly or implicitly, that traditional learning can be enhanced through cognitive science advances and that better education in mathematics can solve all of society's ills and maintain "the American way of life." Less privileged critical theorists would claim that this view of mathematics is in fact the *cause* of society's ills, and that by either overthrowing it or by utilizing its power to harness counter-resistance, society can begin the project of self-reorganization in the image of justice, equity, and authentic democracy. Both of these polemical discourses are implicitly hegemonic, in that they both attempt to garner leadership and power in order to influence the cultural and political dimensions of society (Nielsen, 2003). The addition of a third dimension based on philosophical dialogue may or may not alleviate the tension between two sides of a simplistic ideological framework, but I do believe the added complexity of "thirdness" allows for a strong research agenda to emerge. In addition, identifying the ontological convictions associated with each allows for a more detailed deconstruction of the policy texts issued by either side, and sets the stage for new pedagogical theories for mathematics education to emerge.

By way of conclusion, I would like to briefly review my overarching argument. First, mathematics education reform has historically been flawed, since it has consistently failed to fully understand the philosophical assumptions and theoretical components that inform its own framework. Due to these incoherencies within policy reform efforts, both among the dominant cognitive-objectives school for mathematics education and in critical theory's political objectives, neither axiological objective has come to fruition. In order to create a more cogent set of practices in mathematics education, and to foster not only strong cognitive levels of mathematics understanding in the public school system, but also a potential for praxis to change the world in which mathematics is so pervasive, a pedagogical domain of ontological inquiry needs to be introduced. I wonder not only whether we are teaching mathematics in the wrong way and for the wrong reasons, but also whether this is indicative of the values and assumptions that our society holds about reality and the way in which it functions. This question is, I believe, an important one, since mathematics is so central to our western rationalistic paradigm. Critical perspectives on mathematics education have been too focused on alternative options, without really looking into the fundamental causes that have shaped our current educational system and our society at large. Inversely, dominant perspectives of mathematics education have been reductive and exploitative, serving economic agendas that benefit only a small minority of citizens.

As a university faculty member in an elementary education department, I am often asked why I focus on mathematics education--surely there are other subjects that are more relevant to both public school and wider social reform efforts. Why mathematics of all subjects? My answer to this question is based on the belief that our prejudicial preference for quantification and abstract rational thinking has led to many divisive consequences, both in educational practices and in societal normative values. After all, we can quantify the size of our wallet but not our capacity to love. If we valued our relationships rather than our bank accounts, and our environments rather than our warehouses, the world in which we live might be a very different place. And it is our ontological assumptions about mathematics that influence these values and beliefs. Optimistically, I believe that a reconstruction of the subject of mathematics might help take us out of the socially unjust, environmentally devastated world that we have created, for the very reason that it stands at the center of our current knowledge paradigm and is therefore, at least indirectly, responsible for the current state of our society. Simply, I believe that by changing how we think about teaching this beautiful subject, we may have the capacity to change much more.

Endnotes

1. For a complete historical overview of reform movements in the U.S. see (Woodward, 2004).

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Address Correspondences To:
Nataly Chesky, Assistant Professor in Teacher Education,
State University of New York at New Paltz
cheskyn@newpaltz.edu