For a 'Non-mathematical' Learning of Mathematics. A Philosophical-Educational Reflection on Philosophical Inquiry and Mathematics Classes

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'Αγεωμέτρητος μηδείς είσίτω, that is, "Let no-one without knowledge of geometry enter:" the inscription displayed on the entrance to Plato's Academy reminds us how close the relationships between mathematics[1] and philosophy used to be. In this perspective, when we approach the issue of how philosophical inquiry can further maths' teaching/learning, a sort of archaeological attitude (Agamben, 2008) is in order, which delves into the layers of a long history, plumbs the recondite depths of Western thought, and unearths what remains too often concealed either because it is taken for granted or because we have become unable to detect what constitutes the very way in which we think.

Accordingly, investigating the role and significance of philosophical inquiry for the learning/teaching of maths does not consist simply in reflecting upon the possibility of extending a specific pedagogical model (in this present case the Philosophy for Children approach) to another area of teaching or upon the didactic strategies necessary to do that (pivotal as they are) but, more radically, it means re-thinking the deep solidarity between what we are endeavouring to (re-)harmonize – maths and philosophy.

But is that ancient and venerable harmony between mathematics and philosophy the one which we want to re-establish in our classrooms? Or is the sense in which we understand philosophy when we appeal to its mobilization in math classes profoundly different from that which resonates in the Academy emblem? One thing should attract our attention: the warning on the Academy entrance intimated that math is a pre-requisite for accessing philosophy, a visa to enter the domain of the philosophical. In contrast, by invoking a recourse to philosophical inquiry in math classes, we reverse the order and suggest that through the former we can approach the latter in more profitable, if not more effective, ways. What is entailed in such a change of perspective?

Actually, things are much more complicated; the Academy admonition says not that math is the door through which we can enter philosophy, as if, by studying math, we could gain access to philosophy, but rather that only those who already have a knowledge of mathematics can pass the door which opens onto philosophy. In other words, no one without knowledge of math can hope to be admitted into a philosophical community, but this does not imply that all those who know math will be admitted; math is a necessary but not sufficient condition for philosophy.

On the contrary, by proposing that *through* philosophical inquiry, following the model of Lipman's community of inquiry (Lipman, 1988, 1991, 2003; Sharp, 1987, 1996; Splitter & Sharp, 1995; D. Kennedy, 1995, 1997, 2004a, 2004b, 2012), we can familiarize ourselves with math, we are turning the community of philosophy into a (possible) 'door,' into a way of access. What does it imply for philosophy? Apparently, it is no longer the Academy philosophy, that 'after-the-door,' but another kind of philosophy, but which one?

By questioning the relationships between mathematics and philosophy, within a learning/teaching situation, we are compelled to raise the question of what is philosophy when it ceases to be what is 'after the door' and becomes the 'door' itself, so that the community of philosophical inquiry (CPI) could be characterized, with a grain of irony, as that group which is at the door (of maths?). And, more interestingly, we should ask why this new status of philosophy reveals itself in a particularly clear way precisely when we endeavour to understand how philosophy can be mobilized to learn/teach maths.

Indeed, the texture of this present reflection holds together three topics which include philosophy, mathematics and learning/teaching. Even better, it attempts to bring their unity to light. As I will attempt to show, investigating math means constitutively inquiring into learning/teaching and this, in turn, appeals to an exploration of what philosophy is and what it can be. If Plato's Academy is something more than an experience in Western pedagogy and has become the prototype of every higher education institution, it is because it articulated in an epoch-making way the relationships between mathematics, learning/teaching, and philosophy. And it did it with the force of a commandment (it is noteworthy that the warning on the Academy entrance is in the imperative mode). Questioning this imperative, reversing the order--in any meaning of the word--and making maths and philosophy not two steps in a sort of evolutionary sequence but the allies in an educational project which tries to re-define what learning and teaching are, all this is ultimately what the whole reflection on CPI and maths classes amounts to.

1. Away from the 'Mathematical' Despotism: The Need for a Recovery of Philosophical Inquiry

Why did people have to know mathematics in order to enter the Academy? To capture the meaning of the admonition we have to understand to what end an institution such as the Academy was founded. To put it in a nutshell, the Academy was Plato's response to the scandal of Socrates' death, and it is connected with an overturning of that idea of philosophy of which Socrates had been the embodiment.

In his later studies on Socrates, Gregory Vlastos emphasized that the elenchus (the Socratic method based on questions-answers) is an *inquiry* which, although it aims at truth, does not envision the same truth as that of mathematical reasoning. On the contrary, the 'mathematical truth,' and the specific method connected with it – the 'hypothetical' method which is spoken of in Meno 86e (*ex hypothéseos skopeîsthai, we read there*) –, is what Plato will substitute for the Socratic method in order to obviate the shortcomings of the latter (we will see later what they were). The procedure of Socrates, as Vlastos reconstructs it, is clearly marked in its stages:

> 1) The interlocutor asserts a thesis *p*, which Socrates considers false and targets for refutation.
> 2) Socrates secures agreement to further premises, say *q* and *r* [...]. The agreement is *ad hoc*: Socrates argues from {q, r}, not to them. Socrates then argues, and the interlocutor agrees, that *q* and *r* entail not-p. 4) Socrates then claims that he has shown that *not-p* is true, *p* false. (Ibid., p. 11)

In other words, Socrates operates on the beliefs of his interlocutors, and aims at showing their contradictions. What is most significant in the Socratic method is that it is a real inquiry because the philosopher has no guarantee of success and no absolute ground on which to found his argumentation. As a matter of fact, all steps are negotiated with his interlocutors. This notwithstanding, Socrates is sure that he will always be able to prove the fallacy of the belief p, to stick to the aforementioned example. According to Vlastos, Socrates harbors this conviction not because he believes that he owns a godly wisdom but, on the contrary, because he has only a human wisdom (*anthropíne sophía,* we read in *Apology* 20d-20e), that comes from his previous inquiries, which had borne out the consistency of his own system of beliefs, while "[a]ll others, when tested for consistency, have failed"(Vlastos, 1994a, p. 27).

By drawing upon such an interpretation, and with a certain hermeneutical bending, we could venture the idea that the trust in the consistency of his own system of beliefs is a *presupposition* of Socrates' inquiry *only insofar as it is the outcome of previous inquiries*, without being anchored to any warranty going beyond the inquiry itself. In any discussion Socrates bets on the robustness of his system of beliefs, tests it and exposes it to the risk of failure but, simultaneously, by doing that, he corroborates it whenever he shows the inconsistency of beliefs contradictory with his.

The educational implications of such a procedure are that Socrates' position is not absolute in principle (despite his charisma risking overshadowing this fact): as far as I understand him, appropriating Vlastos' reflections, Socrates is not a doctrinaire teacher but rather one engaged in a real dialogue, and, accordingly, a common inquiry can occur, in which Socrates is also a learner (insofar as he learns that actually his beliefs are the only ones to avoid contradiction). As he does not have any godly wisdom but only a human one, his philosophizing is always a co-philosophizing (*symphilosopheîn*), which is a communal philosophical inquiry animated by a co-educative tension. In this procedure, knowing and educating each other are two sides of the same coin. And such a wisdom is human because it does not pretend to be absolute, it is an inquiry not a quest for certainty, to use a Deweyan (1984[1929]; 1986[1933]; 1986[1938]) vocabulary. But "how could it have happened that each and every one of Socrates' interlocutors did have those true beliefs he need[ed] to refute [from] all of their false ones?" (Vlastos, 1994a, p. 29): this is a question – Vlastos remarks – which never came from Socrates, but Plato attempted to answer through "[t]hat wildest of Plato's metaphysical flights, that ultra-speculative theory that all learning is 'recollection'" (*Ibidem*).

What is most interesting in this context is, however, the reason why Plato felt the need to lay a metaphysical foundation for Socrates' method. The scandal of the trial and of the death of his master convinced Plato that the inquiring method of Socrates, destitute of any metaphysical anchoring, was constantly exposed to the danger of succumbing to its limitations. Plato endeavored to find a kind of 'thinking' which could be a rival in strength with the effective and actual force of the leaders, of the Athenian democracy, to which he had been a witness during the trial. He identified such a 'forceful' kind of thinking in mathematics. Indeed, as Hannah Arendt (1965) remarked, in a completely different context, Plato started the tradition of insisting on:

> the compelling nature of axiomatic or self-evident truth, whose paradigmatic example [...] has been the kind of statements with which we are confronted in mathematics. Le Mercier de la Rivière was perfectly right when he wrote: "Euclide est un véritable despote et les vérités géométriques qu'il nous a transmises sont des lois véritablement despotiques. Leur despotisme légal et le despotisme personnel de ce Législateur n'en font qu'un, celui de la force irresistible de l'évidence;" [...]. In our context it is important to note that only mathematical laws were thought to be sufficiently irresistible to check the power of despots. (Arendt, 1965, p. 193)

As a consequence of the scandal of the trial and death of Socrates, Plato operated an epoch-making shift and what his master had refused (that is, the 'mathematical' research of the *Meno*) is mobilized to give strength to Socrates' thinking. From that point on no one without any knowledge of mathematics would be allowed to enter the Academy. Mathematics became a sort of fortified outpost which protected philosophy from lapsing into the ultimately losing confrontation with the *dóxai*, the opinions of the city. Philosophy is no longer an inquiry investigating the beliefs of subjects but a kind of mathematical thinking, which derives consequences from first principles which are self-evident.

In this new thematic constellation, at least as I am interpreting it, the alliance between mathematics and philosophy occurs at the expense of inquiry understood as a kind of research which has no guarantee, no axiomatic certainty, no absolutely firm ground. It is important to bear in mind this fact because here we find both the reasons for the mistrust that the proposal of using P4C for maths classes can encounter and those for a possible failure of such an educational proposal, when it is not clear to which idea of philosophy we are referring. Indeed, on the one hand, to the extent that math continues to be considered the paradigm of 'compelling reasoning,' any mobilization of P4C could appear to be inappropriate, if not detrimental, insofar as it risks 'infecting' with discussion and dialogue what should be intrinsically monological because absolutely certain. On the contrary, what will be argued here is that there is – in a quasi-Deweyan sense (1980[1917]) – a need for a recovery of mathematics which is parallel and even interwoven with the need for a recovery of philosophy, to which Lipman much more than Dewey gave educational expression (Oliverio, 2012b).

On the other hand, this recovery is possible only if we are clear about what kind of philosophy we are speaking of and about the radicalism of Lipman's proposal of the community of philosophical inquiry, which represents a counter-movement in comparison with the Academic one. To put it in a slogan: through P4C and CPI Lipman realizes, at an educational level, a 'back to Socrates' (Lipman, 1988, p. 12) movement, Socrates being the name for the search for a 'human wisdom' as opposed to the *ex hypothéseos skopeîsthai* characterizing mathematics.

As the Socratic method, as far as I have presented it in Vlastos' wake, predominantly addressed ethical issues, it could be suggested that P4C and CPI can and should be used in math classes only when we are interested in exploring the moral and political dimensions of mathematics. While it is plausible that this represents one of the opportunities offered by P4C applied to maths classes (N.S. Kennedy, 2012a), it does not exhaust the range of possibilities. We should understand in what sense Socrates' inquiry is an ethical one. As far as I construe his undertaking, Socrates attempts to bring his interlocutors to ethics passing through an examination of morals. 'Moral' comes from Latin *mos* and refers to the mores of a society, to that set of rules which are codified and are often taken for granted and complied with without any personal commitment. In contrast, 'ethics' comes from the Greek *éthos*, which originally means 'the appropriate place.' The 'ethical' inquiry properly understood does not deal (only and primarily) with morals but with how the individual positions him/herself in relationship to the world, and it concerns, consequently, the whole being-in-the-world of individuals, their existence and, more particularly, the ways they find something existentially meaningful. 'Ethical inquiry' is a search for meaning and, from this perspective, the entire Lipman and Sharp enterprise, insofar as it is directed to meaning (Lipman *et al.*, 1980), is 'ethical' and therefore 'Socratic.' Accordingly, engaging with mathematics through ethical-Socratic inquiry (= Lipman philosophical inquiry) adds up to more than investigating the moral or political dimensions of math. It rather concerns the question of to what extent math is meaningful, how it can be and how individuals make sense of it. It is a search for meaning while learning math: in this perspective it is 'ethical.' It allows individuals to find 'the appropriate place' for math in their existence. And this happens not only at a strictly moral but also at an 'epistemological' level (see below § 3).

Before investigating in more detail the educational and pedagogical implications of this shift, I want to explore what the philosophical-educational characteristics of the Academy are so that the peculiarities of the CPI approach can stand out more clearly.

2. The Overthrow of the Academic Model and the 'Pragmatic' Learning

In describing the Academy, I will focus not on the historical debates occurring there (which still represent a paradigm for intellectually open discussion [Berti, 2010]) but rather on the quasi-archetypical image of it which has congealed in the Western tradition. The exploration of this image will allow us, on the one hand, to identify the differences of Lipman's (and more generally a pragmatist) understanding of philosophical inquiry in comparison with the traditional educational model and, on the other, to highlight how the predominance of 'mathematical' despotism affected the Academic view of philosophy.

In the Academy, the reality of philosophy consists primarily and essentially in the philosopher's vision of the principles, the absolutely metaphysical ideas, that constitute the very core of the world. This vision is not the outcome of a common inquiry but of the 'intimacy' of the philosopher, in his solitude, with the object of his theory. The Seventh Letter (341b-341c) bears witness to such a view of philosophy. Plato opens up a gulf between the 'theoretical activity' of the philosopher and teaching; the former cannot be taught *stricto sensu*, teaching will always be not only a derivative activity but one that cannot convey what is the innermost nature of theory. Consequently, also any cooperation is excluded: inquiry is solitary, it is hardly inquiry in an appropriate sense; as a matter of fact, it is vision (*theoria*, in Greek).

Against this backdrop, the imperative on the entrance of Academy is comprehensible: the paradigm for such a theoretical activity is the mathematician isolated from the world of worldly appearances and immersed in the universe of numbers (purely 'abstract' entities, the intercourse with which is an isolated mental matter and excludes any inter-human intercourse).

Hierarchically subordinate to this first reality is what Paul Landesberg (1923) calls "the second reality of philosophy," which is constituted by the disciples of philosophers and propagates in the successors of philosophers during the ensuing centuries. The writings of philosophers are one of the means of spreading this second reality but they "are in no way a reality of philosophy but precisely only printed or written paper" (Landesberg, 1923, p. 95). The genuine life of philosophy is realized in the vision, which belongs to the philosopher and is then (partly) communicated to disciples and successors. In the Academy device, while the first level reality can exist without the second level (= the innermost core of philosophy – the vision of the thinker – does not need a community of co-inquirers), the second is nothing without the first.

It is important to note two things: first, in this perspective, the circle of disciples is only a bridge between philosophy in its real essence and its socialization and dissemination. Students are not a part of the very reality of philosophy, and do not participate in the production of thoughts but only in their communication. And, second, their function is, though, superior to that of the writings because "philosophy is real only when it is realized and taught: in philosophizing" (*Ibidem*). To put it schematically:

- Level 1 = The isolated philosopher's vision/theory = The innermost reality of philosophy;
- Level 2 = The circle of disciples = The communicated reality of philosophy;
- Level 3 = The writings = The means of communication, therefore destitute of any real bonds with the reality of philosophy (indeed, they are connected with Level 2, but as the connection is only instrumental, they do not affect in any way the reality).

With this model in mind, we can assess the novelty of the idea of a classroom turned into a community of philosophical inquiry. In CPI, philosophy does not exist prior to and outside the circle of students who co-philosophize. And students are not disciples of a philosopher, whose vision they rehearse or re-cite, but they are the producers of thoughts within a plural setting (Level 2 > Level 1). Philosophy is here not the outcome of an isolated soul/mind immersed in the contemplation of ideal entities but of a distributed thinking occurring in a space-temporal context. And written texts are not the soulless 'materialization' of philosophy but, on the one hand, under the form, for instance, of 'philosophical novels,' they are what triggers the philosophical inquiry, without which no philosophy would exist, and, on the other hand, – under the form of the agenda on the paper-board – they are what embodies the distributed thinking of the CPI (and, consequently, they are not only written paper but the 'objective correlate' (in T.S. Eliot's phrase) of the philosophical activity). Thus, the Academic schema is completely reversed.

It is crucial, however, to highlight that, thanks to the CPI approach, it is not a mechanical reversal which we have to do with here, but rather a real deconstruction, that is, in a quasi Derridean vein, something that activates and operates on what remained unsaid and concealed in the Academy model. This is the case in at least two respects: first, in the Academy model, as disciples occupy a middle position (between the vision of the thinker and its social dissemination) and as they constitute what 'actualizes' philosophy because philosophy is humanly actualized only when it is taught (see Landesberg, 1923, p. 95), they are a sort of intermediary and have, then, precisely the mediating position of the philosopher, such as Pierre Hadot (2002) magnificently depicted it by comparing Socrates and Eros. The real philosophy in the 'Socratic' sense resides not in the self-secluded soul/mind of a philosopher en-visioning first principles but in the co-educative dynamics of a communal inquiry. There is no genuine philosophy if not within a co-philosophizing as educational relation.

Secondly, when speaking of writings and condemning them in comparison with the first reality (the spark in the psyche of the thinker), Plato uses an interesting word: *súggramma*, that is, the semantic root for writing (*-gramma*) and the prefix *sun-* (=with). The Platonic textuality suggests that the writing and the co-operative dimension of philosophy as an inquiry are intimately interwoven. And if, as we can state through a hermeneutical twist of Landesberg's text, writings – insofar as they are re-actualized in that wordly realization of philosophy that its teaching – are "the lasting power of [the] birth" of philosophy (Landesberg, 1923, p. 95), then it is within a session of philosophical inquiry, set off by a philosophical novel and – possibly – culminating in another written text (the written agenda), that philosophy comes into the world over and over again. Better, philosophy is nothing but a continuing re-birth within the context of a communal inquiry made possible by writings and animated by an educational tension. The static 'mathematical' contemplation is replaced, then, with a movement of re-birth.

A second dimension of 'mathematic,' one which is related directly to learning, needs to be investigated. In order to define 'mathematics' Heidegger (1987) refers to *tà mathémata*, the things insofar as they are learnable, from which he distinguishes *tà prágmata*, that is, the things insofar as they are that with which we have to do, to deal with, something related to *práxis* understood as any human doing. If *tà mathémata* are the learnable things, what is learning? According to Heidegger, learning is a kind of acquisition, a 'taking':

> The *mathémata* are the things insofar as we take cognizance of them as what we already know in advance [...]. Such a proper learning is thereby an extremely remarkable taking, a taking where who takes takes only that which fundamentally he already has. (Heidegger, 1987[1936], p. 56)

What the student already has and is, therefore, learnable, 'the mathematical' in Heidegger's sense as I am idiosyncratically reading him, is what is alien to the existential level and belongs to the ideal realm, that is, to that domain accessible only to a disembodied, self-secluded reason.

What happens when such a kind of learning, which treats things 'mathematically,' that is, as learnable things (=ultimately known in advance, in the sense that they are cogently valid and are to be *demonstrated* and *not discovered through an inquiry)*, is replaced with a 'pragmatic' learning, that treats its contents (even the specifically mathematical ones, that is, numbers, etc.) as something which we have to do with? If we can construe the entire Lipmanian undertaking 'pragmatically,' that is, as a way of putting philosophy into practice, of educating for *doing* philosophy (Lipman, 1988, p. 12), the issue we should finally investigate is what happens when this way of understanding philosophy is used for math classes, for teaching precisely that subject-matter that seems the least susceptible to any 'pragmatic' learning, and so much so that it is the very paradigm – in the Academy – of the flight from existence into the domain of the abstract.

In § 1, I have spoken of an 'ethical' approach to mathematics, one that explores, through philosophical inquiry, the meaning of mathematics within one's own existence; in this paragraph, the idea of a pragmatic learning was put forward. It is time, now, to outline how the deployment of philosophical inquiry can act on math classes in an 'ethical-pragmatic' perspective.

3. The Tears of Philippa: Philosophical Dialogue and Math Classes

In his obituary for Rorty, the Italian mathematician Giorgio Bagni (2007) pointed out the significance of Rorty's philosophy for math education by insisting on his anti-Platonism:

> As a matter of fact, a Platonic approach cannot be stated uncritically in educational practice [...] The connection between knowledge and social practice is really a crucial issue from the educational point of view [...] Richard Rorty strongly underlined the crucial importance of the community as source of epistemic authority [...]. (p. 2)

Bagni's remarks are interesting because they allow us to outline the specificity of a Lipmanian approach to math. To say it with a formula, we could state that Lipman occupies a middle ground between Plato and Rorty, avoiding the shortcomings of both approaches. While it can be maintained – and this has been the topic of the previous paragraphs – that a non-Platonic stance towards philosophy and mathematics can represent a major gain for math education, it is moot whether a typically Rortyan emphasis on solidarity instead of objectivity (Rorty, 1991) would represent real progress in math education.

In other words, while Bagni appears to be enthusiastic, in his few lines praising Rorty's philosophy, about the prospects that the latter opens up to math education, I would like to highlight that Rorty's stress on community is profoundly different from Lipman's and that, while the latter can add to the meaning of math learning/teaching, the former risks dissolving it. Indeed, due to his misgivings with the notion of inquiry, it is debatable that Rorty would accept serenely the idea of a community of philosophical inquiry. He would have probably found it still too mortgaged by an 'objectivity-oriented' attitude, as if the value and the raison d'être of the community were external to it and to the solidarity-principle. As was sagaciously remarked (Silva, 2010), Rorty and Lipman represent two misreadings, in a Bloomian (1973, 1975) sense, of Dewey's legacy and I would tend to believe that, while Lipman can play a major role in reconstructing the practices of math education, Rorty can provide us with helpful suggestions for the *pars destruens* but can have little import on the *pars construens*.

Indeed, the core of the Rortyan insistence on solidarity is that only dialogue counts and not also what the dialogue is directed to. The peril I see in this perspective is that it represents only the antipodean opposition to the Academy model I have depicted. If in the Academy community counted for nothing, as far as the theoretical searching for truth was concerned, in Rorty's device the search for truth counts for nothing. The notion of "conversation" captures beautifully this change of perspective. If it is obviously possible that within a solidarity-oriented community a continuous adding to the repertoires of meanings occurs (indeed, it is the mark of a flourishing community), this is not the outcome of an inquiry understood as a search for truth.

On the contrary, as Susan Gardner has magnificently argued, "progress toward truth is vital to the practice of inquiry and [...] if such progress is not made, the term 'Community of Inquiry' becomes a misnomer" (Gardner, 1995/1996, p. 102).

The pedagogical repercussion of these distinctions (conversation ≠ dialogue; Rortyan community ≠ Lipmanian community) is that in a conversational approach looking over the development of the inquiry during a class (or a P4C session) can be overlooked – let the pun be allowed – because the mere progress of conversation is enough and can contribute to the strengthening of the solidarity of the community. In contrast, in the CPI it is crucial that the glue of the community, that is what holds the latter together, is the commitment to a search for truth (leaving, for the moment, aside a more clear characterization of this truth). Ignoring this search is calamitous for a P4C session (many of us are painfully aware of how what goes on in classrooms is too often not a P4C session but a nice conversation starting from Lipmanian texts and sticking to some Lipmanian procedures). And this would be even more the case, if we wanted to mobilize this conversationally-weakened kind of CPI for math education.

The search for truth, without being a capitulation to a 'Platonic objectivity,' supplies the community with an inquiring horizon, which should structure the relationships occurring in it and give the dialogues a direction (what Gadamer (1960) would call the *logos* of the dialogue). The teacher oversees such a direction by contributing to the harmonizing of the different strands of the dialogue (corresponding to the different positions of the students, which are not to be reduced to uniformity) in a dialogic-unity-out-of-differences.

I am insisting on this idea of a dialogue directed to (a non-Platonically-objective) truth to prepare a conceptual platform for the discussion of the role of philosophical inquiry within math classes, by putting in relief both the ways in which this approach is linked with other experiences in math education and its peculiarities, which, in my opinion, allow it to eschew some drawbacks.

Paul Ernest (1994) focused on the dialogic nature of mathematics itself, showing how it "sits at the crossroads of two major currents of modern thought, the recent fallibilist tradition in the philosophy of mathematics, and the multidisciplinary use of the conversation as a basic underlying metaphor for human knowing and interaction" (p. 33). As to the first current, its main feature consists in the rejection of the following four theses:

> 1. There is a secure and fixed basis of truth on which mathematical knowledge is founded;
> 2. There are wholly reliable logical deductions of mathematical theorems from explicit premises;
> 3. Absolute mathematical knowledge based on impeccable proofs is an ideal which is attainable;
> 4. The logical properties of mathematical proof alone suffice to establish mathematical knowledge without reference to human agency or the social domain.
> - These theses underpin the traditional absolutist views of mathematical knowledge and establish its monological character. They are also central assumptions of Cartesian rationalism and the modernism based on it. (Ibid., p. 35)

It is true that these theses belong to modern rationalism but, as I have tried to illustrate above, they have their most ancient forebear in the Academy model.

As to the second current mentioned by Ernest, he summons different thinkers such as Rorty, Wittgenstein and Gadamer. In order to launch an alternative to the monological tradition in math education it could be appropriate to display such a panoply of philosophies, yet I would tend to consider the 'conversational' strand of this new dialogic tradition less promising for math classes. Even if we take leave of the absolutist view, it is important to bear in mind that inquiry, also within mathematics, demands a direction to truth and, therefore, that some 'dialogic philosophies' are more suitable than others in order to underpin a renewal of math learning/teaching.

Ernest shows also how the dialogic nature of mathematics "encompasses its textual basis, some of its concepts, the origins and nature of proof, and the social processes whereby mathematical knowledge is created, warrented and learnt" (Ibid., pp. 44-46). Apart from the misgivings about the idea of conversation, I agree with the main point made by Ernest. But if his argumentations permit us to find, within the domain of mathematics education, a respondency to the (philosophical-educational) reflections conducted in this present paper, they do not constitute per se the 'proof' that something like *philosophical* inquiry/dialogue is beneficial for math classes.

To put it in the P4C vocabulary, my discussion plan will be finally the following: if maths education should always have a dialogic nature (in keeping with the dialogic nature of mathematics itself), when, why, to what end and in what forms is a *typically philosophical dialogue* helpful?

Over the last years Nadia Stoyanova Kennedy has been investigating such issues and has spoken of "interruption," by drawing upon a Biesta (2006; 2010) notion. What is interrupted is the "normal order" ruling over the pedagogical praxis in classrooms and the interruption "may be as straightforward as prompting students to question their own understandings of a concept under discussion, to reflect on what they know and what they do not know, to question their peers' understanding of the concept" (N.S. Kennedy, 2012a, p. 261).

So understood, interruption is a quasi-Socratic, torpedo-like move, which prevents students from falling into a 'disciplinary' slumber, that is, into the risk of taking for granted the concepts of a discipline (as Socrates' fellow-citizens took for granted the mores of their city). And it permits individuals to reconnect the learning of the discipline to the broader context of their own being-in-the-world (the ethical dimension of learning math, in the sense spelled out above).

It is important to appreciate the peculiar note of the notion of "interruption:" it does not refer to a breaking-in from without, but to something that, by operating on the interstices of the bodies of disciplines, breaks the spell of the monological closure – always looming over every discipline – and opens up a space for discussion and dialogue. This kind of inter-ruption is the condition for the 'interest,' understood etymologically as the being in-between, at the door, to use once again the metaphor I have started with.

If it opens up a space for dialogue, which reconnects math in its dialogic nature to human transactions with the world, philosophical inquiry represents also a radicalization of the inquiring nature of mathematics itself. John Mason (2002, p. 109) has beautifully spoken of a "conjecturing atmosphere" which sustains learning of mathematics. And in an analogous vein, Derek Holton (1997), by building upon Legrand's idea of the *débat scientifique,* remarks:

> Under le débat scientifique, [...] students are seen as participants in a scientific community whose methods of development include conjectures, proofs and regulations. Scientific debates can arise spontaneously, as when a student asks a question, or can be intentionally provoked. The guiding principles for scientific debate include:
> - disturbance – students must encounter and deal with conflict;
> - inclusiveness – everyone should have an opportunity to understand what we try to teach; and
> - collectivity – collective resolution of issues shows how to work with contradictions and to respect the views of others.
>
> Now it may seem strange that what is labelled "scientific" has such a strong social underpinning. Maybe this can be explained by noting that the point of the exercise is to allow students to engage in "scientific debate." This requires an atmosphere where conjecturing is supported, where students feel free to put forward their ideas, where they are not embarrassed to make a mistake, and where they feel that they are able to modify the ideas of others. (p. 4)

I have quoted at length this passage because it refers to a socio-epistemic dynamics very similar to that in the CPI: *what is, then, the specificity of the CPI?* If the philosophical dimension has to be more than a content-ingredient, it should represent rather a difference in that dynamics.

To capture this point we have to refer back to the very idea of the community of philosophical inquiry. The phrase was drawn by Lipman from Peirce (D. Kennedy, 2010, p. 15), but with a major qualification, expressed by the adjective 'philosophical.' In Peirce, inquiry is stirred by a "genuine doubt," that is, by something that intervenes to disrupt our beliefs. Genuine doubt is some real "indecision, however momentary, in our action" (to use the Peircean expression), which is obviously quite different from the kind of interruption which philosophical inquiry produces. The whole project of a community of philosophical inquiry would be, therefore, ultimately, *pace* Lipman, non-Peircean, and disconnected from the way a scientific community works and unable to have effects on inquiries concerning science (and also math, for that matter).

But things are, actually, more complicated and by understanding in what sense Lipman is really faithful to one dimension of the Peircean legacy we can appreciate also the contribution which CPI can provide maths classes with. As Browning (1991) noted, by 1905:

> Peirce recognized a form of philosophically significant inquiry which did not have a starting point of genuine doubt. [...] The preferred form of philosophical investigation is [...] that which serves both to lead towards and encourage genuine doubt and to proceed, once such doubt has been so brought about, to its destruction by belief. But on this view the starting point of philosophical investigation is no longer genuine doubt, which now occupies a middle point in the investigation, but something quite different. This new starting point, though not adumbrated in any detail by Peirce, appears to consist in or be instituted by a sort of voluntary act in which one "sets himself" to reflect upon and examine certain of his beliefs. (pp. 20-21)

The ingenuity of Lipman consisted in the fact that, without presumably having any accurate knowledge of Peirce's thinking, he was able to grasp one important element in the latter's epistemology and to translate it into a powerful educational device, that is the CPI. When used for the learning/teaching of disciplines, the CPI is the pedagogical approach which, by soliciting philosophical doubts (step 1), can contribute to the emergence of 'genuine doubts' (step 2), without which no true inquiry (step 3) can be realized.

Indeed, it is often insufficient to promote courses of (non-philosophical) inquiry, which may remain incapsulated in the matrix of the discipline. It is obviously welcome to use any kind of teaching strategy, which prevents students from lapsing into the trap of what Leibniz called psittacism, the parrotism in the re-citation of lessons. But not always, despite their merits, can inquiry-based methods obviate adequately this risk. As a matter of fact, if not generated by a genuine doubt, the inquiry which develops, brilliant and interesting as it may be, can be less fruitful in terms of a real understanding of the topic than expected. The students risk playing more the role of Kuhnian (1970) puzzle-solvers than that of Peircean inquirers. Consequently, a significant move of the teacher could/should be that of promoting the emergence of a genuine doubt via the mobilization of philosophical inquiry, according to a Peircean-Lipmanian model.

I will provide a short example of what I mean. In a 4th grade class in Italy pupils had to order the following measurements in an ascending order: 7.50 dm; 8.1 dm; 7.8 dm; 7.09 dm; and 8.15 dm. And the following dialogue took place (see Sorzio, 2013, pp. 143-144):

1) Tommaso: "Seven point nine is smaller ... because of the zero."
2) Giulia: "Because the zero does not have any value, you can then say 7.9."
3) Andrea: "But there is also 7.8."
4) Tommaso: "That's true, '7 point 9' because nine is a millimetre."
5) Teacher: "Who agrees?"
6) All: "Yeahhhhh!"
7) Teacher: "Give me then a good reason."
8) Silvia: "Because 7 is a decimetre, 0 is a centimetre, 9 is a millimetre."
9) Teacher: "0 is not a centimetre, it is zero centimetres. And 7.8, what is it?"
10) All: "7 decimetres, 8 centimetres."
11) Teacher: "Then, why is 7.09 smaller, can you explain it well?"
12) Silvia: "Because centimetres are bigger than millimetres."
13) Giulia: "No!"
14) Silvia shows centimetres and millimetres on the ruler.
15) Teacher: "Why is it smaller?
16) Tommaso: "Because it has the millimetres ... because the zero does not have any value ... it is the zero that indicates ..."
17) Teacher: "No, no, it is not because the zero has no value ... it is because the zero indicates ..."

18) All: "The centimetres."
19) Teacher: "7 refers to the decimetres, the 0 tells us that this measurement is 0 centimetres and 9 millimetres, and the other is 8 centimetres, and then ..."
20) Tommaso: "7.9."
21) Silvia: "7.09."
22) Teacher: "And then, what is the following number?"
23) Tommaso: "7.8."
24) Teacher: "Are you sure?"
25) Giulia: "Yes, because we take a small step back."
26) Teacher: "No, reflect."
27) Andrea: "7.50."
28) Teacher: "Why?"
29) Andrea: "Because 5 centimetres is smaller than 8 centimetres."
30) Teacher: "Who agrees? Who disagrees?"
Many disagree.
31) Tommaso: "5 is smaller than 8 because the 0 doesn't have any value."
32) Teacher: "You should not see the zero as smaller but as something that indicates a smaller element."
33) Silvia: "This 0 should be 0 millimetres; 0 millimetres is not on the ruler and therefore millimetres are not there because if it is 0 they are not there."
34) Teacher: "So, 8.1 and 8.15, which is the smaller?"
35) Tommaso: "8.15 is smaller because here we have the millimetres."
36) Teacher: "And for this reason is it smaller?"
37) Tommaso is perplexed and some disagree.
38) Andrea: "8.1 and 8.15 would be 8 dm and 1 cm., and 8.15 8 dm, 1 cm and 5 mm [the others agree] here it is only one centimetre and here it is in advance by 5 mm."

The dialogue here is often on the verge of turning into a sort of puzzle-solving. The 'logic' of the interruption could suggest, instead, that at move 17) the teacher could have 'expanded' her intervention up to a philosophical level and opened up the space for a different kind of questioning, instead of almost intimating the right answer, by referring to the words of Tommaso at 16). Before going back to the mathematical problem, she could have asked: what is the 'value' of zero? What do you mean by saying that zero has no value? Are zero and nothing the same thing? When we have zero, do we refer to nothing? What is nothing? Is it the same as zero?

It is apparent that one of the issues of the dialogue (although not the only one) concerns the idea of zero having a/no value, and that this represents a sort of "epistemological obstacle" (Bachelard, 1938) to fully understand how to order the measurements. Staying only within a purely mathematical framework, that is, within a purely intra-disciplinary discussion, could lead finally to the right answer but without a full 'grasp' of the concepts. The philosophical interruption could offer the chance to fine-tune the understanding of some concepts and to establish a theoretical platform to go back to the mathematical inquiry with more awareness.

The New Zealand mathematician and math educator Bill Barton invented an interesting story, included in his book *The Language of Mathematics*, which aims at "argu[ing] that mathematics is a human creation [and at] show[ing] that in the origins of mathematics humans had the opportunity to create it differently that (*sic*!) they did" (Barton, 2009, p. 73). He builds on notions with which I would tend to agree, such as, for instance, the central role of communication and language (see also Devlin, 2000), the importance of the metaphor and the embodiment (see also Lakoff & Nunez, 2000), and the need to reconnect mathematics and experience, but he gives these notions a radical 'spin,' which, in my opinion, is too affected by some strands of postmodern thinking and epistemology, while I would rather suggest situating them within a Deweyan, post-postmodern (Hickman, 2007; Oliverio, 2013) framework. But this is a line of discussion which exceeds the scope of this paper.

In the fictional story (Barton, 2009, pp. 73-77) a teacher sets this problem: '1/4 + 3/8 = ?' and receives four different answers:

- Johnny: 1/4 + 3/8 = 4/12
- Mere: 1/4 + 3/8 = 5/16
- Tom: 1/4 + 3/8 = 3/32
- Philippa: 1/4 + 3/8 = 5/8

Barton is brilliant in describing how Johnny, Mere and Tom provide ingenious explanations (ultimately rooted in the possibility and legitimacy of other kinds of mathematics), and, consequently, in driving home his point that "[t]he four ways of 'adding' (that is, combining) fractions are all valid in their contexts" (p. 77), because mathematics is language- and culture-sensitive and, for this reason, a teacher "may be more careful about using the words 'right' and 'wrong,' preferring rather to mention conventions more often, or to explain the context of mathematical concepts" (*Ibidem*).

As I have already said, I would not subscribe to the radicalism of his proposal, but I am interested here in what happens to Philippa, the only pupil who had given the 'right' (*pace* Barton) answer. She is invited to explain why she got that result after the explanations of her classmates:

> Philippa refused to move. "I'm the only one who got it wrong," she sobbed. "*I thought I had learned the correct method, but when I look at it, it makes no sense.* I can understand writing one quarter as two eighths, but there is no reason to add only the top numbers and not the bottom ones. That doesn't seem right. Why would you do that? *All the others have got a reason for what they did, and I don't." Philippa learned the method the teacher had taught the class, but had no example to illustrate it, and no rationale for her technique*. She was bright enough to understand the methods the others had used, and they made sense for her. *Her own method made no sense,* and no matter what the teacher said, Philippa was too deeply embarrassed to be comforted. (Barton, 2009, pp. 76-77. Italics added)

While Johnny, Mere and Tom are ready to give reasons, because their answers are ultimately grounded in their experiences and in their worlds-of-life, Philippa does not succeed in providing *that* kind of justification and, consequently, she is cast into despair and begins nurturing doubts about her proficiency in maths (finally, she thinks she is wrong, although she should know that the calculations she made were right). Opposite to the author's intention, I interpret this story not as a parable on the plurality of the mathematical worlds but rather as an illuminating apologue about how much an 'existential' understanding of mathematical procedures, concepts, methods, etc. is needed, if we want to have an accomplished knowledge of maths. This requires a 'non-mathematical' but 'pragmatic' learning of mathematics, to use Heidegger's vocabulary. There is the need to refer mathematics back to the *Lebenswelt* (Husserl, 1959), in order to contrast the "alienation of the sense" of the mathematical formulae (Ibid., § 9f) and their mechanization (Ibid, § 9g). What is – in Husserl – a grand narrative about the destiny of Western civilization recurs on a smaller scale in every class.

In Barton's story, the teacher postponed until the ensuing lesson the explanation to the class of why Philippa was 'right' and "felt the wave of relief as the bell rang, and went off to rethink what it was that she was doing in mathematics" (Barton, 2009, p. 77). The teacher was unable to find any word of comfort for Philippa. And, surely, drawing upon her pedagogical armory in math teaching risked being ineffective. The tears of Philippa denounced not a lack in proficiency in adding up fractions but in making sense of this. And, moreover, how could the teacher foster a really transformative learning (in a quasi Mezirowian (1991) sense) in Johnny, Mere and Tom and not just confine herself to inflicting a mathematical explanation on them without conveying the sense of why they should abandon their theories grounded in their experience? Without promoting philosophical inquiry, the teacher might never be able to stop Philippa's tears.

Endnotes

1. While the motto on the entrance to the Academy refers to geometry, it can be extended to mathematics too. In this present paper I will consider the expression "´Αγεωμέτρητος" as equivalent to "the one who does not know mathematics." I can not provide here a more specific justification of such a move (see Husserl, 1959).

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