Make Believe Numbers: A Multidimensional Analysis of Phillip Pullman's Mythopoeic Vision

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> "But think of Adam and Eve like an imaginary number, like the square root of minus one: you can never see any concrete proof that it exists, but if you include it in your equations, you can calculate all manner of things that can't be imagined without it" (Pullman, p. 372).

The paragraph above, quoted from the end of *The Golden Compass* (*Northern Lights* in the UK) seems to encapsulate not only Philip Pullman's spiritual worldview, and his evangelization of that view to adolescent readers, but also, as we shall see, his, perhaps, incomplete understanding of the concept of *dimension*. This is significant, because Pullman's view of the cosmos is undoubtedly multidimensional.

There has been a great deal of controversy over Pullman's work, partly, it would appear, generated by himself and his publishers, but, it must be added, the controversy has been enjoined by various Christian denominations, notably the Roman Catholic Church. Pullman is an atheist, but the *His Dark Materials* (Pullman) books are not atheistic; they are anti-theistic and anti-religious. In particular they are anti-Christian. All the villains are part of Church establishment. Officially, the Catholic Church seems to be of the opinion that the books are specifically anti-Catholic, although it is difficult to see where this contention comes from. The first book in the trilogy is set in an alternate universe, contemporary with ours, but with a technology that seems, in some ways, to lag about a century behind. Centuries in the past there has been a Reformation, culminating in the pontificate of John Calvin. At the time of the story, there is no longer a Pope, only a governing body known as the Magisterium. All Christendom seems to be part of this one reformed church. The Christianity portrayed in this book, however, bears no specific resemblance to Roman Catholicism except for the word "Magisterium", the Catholic term for Church Authority, and a closer analysis reveals that this term refers not to authority as a concept but to a specific governing board, which is not the Catholic understanding of the word at all. The word is the same, its meaning is different. Indeed, what little description there is of Christianity in Pullman's alternate universe bears more resemblance to what an Anglican might term "low church." Be that as it may, there is controversy, which is good for book sales.

Pullman does have an antipathy to Catholicism, which is not to be found explicitly in his trilogy, or the use of the term Magisterium, but, in a November 2007 interview with Peter Chattaway, it is quite clear that he opposes authoritarianism and theocracy. As he says, in talking about JRR Tolkien's Catholicism, "Tolkien was a Catholic, for whom the basic issues of life were not in question, because the Church had all the answers. So nowhere in 'The Lord of the Rings' is there a moment's doubt about those big questions....Enormous as it is, TLOTR is consequently trivial."

Interestingly, although Pullman is an atheist, God does exist in these books, as the arch villain – or, perhaps, more accurately, an ex-arch villain who is now a decrepit figurehead for the new Heavenly Powers – and, at the close of the trilogy, the multiverse is saved when God is killed. Why would an atheist want to depict the death of God? How can God be killed if God does not exist? Clearly Pullman is, to some degree, speaking metaphorically. As a matter of fact, in the interview with Peter Chattaway, he says "I revel in the ambiguities and shadows

and suggestions of metaphor." In other words, perhaps, it is not so much God that Pullman seeks to kill as the belief in God. Maybe the Catholic Church is right, after all, to read Pullman's Magisterium as a metaphor for its own. If Pullman writes metaphorically, we should look for the metaphor in the paragraph quoted at the beginning of this essay.

Lyra Belacqua, the adolescent heroine of the *His Dark Materials* trilogy, is told that she could never see any concrete proof that imaginary numbers exist. Pullman seems to be saying "Imaginary numbers (things spiritual) cannot be proven to exist and, therefore, they do not exist." This is a materialist position that, apart from being an obvious fallacy, displays something rather self centred and arrogant in the assumption that any existence in the world is predicated on our observance and acceptance of it.

In the Chattaway interview, Pullman even goes so far as to insist that God is a metaphor:

Perhaps it might be clearer to call him a character in fiction, and a very interesting one too: one of the greatest and most complex villains of all – savage, petty, boastful, jealous, and yet capable of moments of tenderness and extremes of arbitrary affection – for David, for example. But he's not real, any more than Hamlet or Mr Pickwick are real. They are real in the context of their stories, but you won't find them in the phone book.

In the early Twentieth Century, Gödel demonstrated that there would always exist statements, in any sufficiently sophisticated system (Arithmetic being his example), that could not be shown to be true or false. He did not show that the statements were neither true nor false, merely that their truth or falsity could not be proven.

Statement: Imaginary numbers exist. Perhaps this is one of Gödel's statements, one that we cannot prove or disprove. This does not mean that such numbers do not exist, or that they do. To be rather less mathematical: for years I kept my telephone number unlisted. I could not be found in the phone book. I am fairly confident that I existed through those years.

As with imaginary numbers, so with things spiritual: just because we may never find a "concrete proof" that such exist, this does not mean that they do not exist. We can only know that they do not exist when we are given that proof. Proof would, in any case, seem to be a moot point, since we have none in either direction. For either side of the theist/atheist debate to imply that the other is in error because they have no proof is to hoist themselves on their own petard.

The above musings, however, turn out to be purely academic since, as will become apparent, we actually do have a proof of the existence of imaginary numbers, one that is physical, if not concrete. What is an imaginary number? The definition that most of Pullman's adolescent readers will have been given is that imaginary numbers are the square roots of negative numbers. Almost as soon as a young student has been given this definition in a mathematics class, she will be told that such things do not exist. If any real number (positive or negative) is multiplied by itself, the answer is positive, and, since all squares are positive, only positive numbers can have square roots. Our student is, then, perfectly justified in asking what the point is in studying something that her teacher has so ably demonstrated cannot possibly exist in the first place. Perhaps she is given no justification at all – I suspect that this is all too often the case – but, even if she is, the justification is likely to be something along the lines of an argument that, by incorporating imaginary numbers into our way of thinking, we are able to make discoveries that would be difficult or impossible to make otherwise. Our student is told about imaginary numbers precisely what Asriel tells Lyra.

While it is certainly true that the use of complex numbers (sums and differences of real and imaginary numbers) does lead to demonstrations of such results as the fact that the number of primes less than or equal to a large number is approximately that number divided by its natural logarithm, it is not at all clear that such facts may not always be demonstrated without the use of anything imaginary. Moreover, such demonstrations are university level mathematics, and so this justification for accepting imaginary numbers demands an unfairly long

suspension of disbelief on the part of our student.

Perhaps the reader is wondering why so much fuss is being made about a comment made to a fictional character in a work of pure fiction. Internal self consistency would appear to be the only criterion that needs to be considered in this instance. At this point it would be appropriate to point out that Lyra, when she is told about the link between imaginary numbers and Adam and Eve, has had, according to Pullman, a very inadequate education indeed, one that she has done everything in her power to avoid. The chances that she has ever heard about imaginary numbers, or even that she understands the concept of square roots, are not very great. On the other hand, most adolescent readers in the Western world have reached this point in the Mathematics curriculum. Lord Asriel's conversation with Lyra is not consistent with the story to this point. Nor is it consistent with the rest of the story; Asriel does not disbelieve in things spiritual, and he is, in fact, fairly convinced of God's existence. It is his ambition to kill Him. It is apparent that the comment is actually external to the narrative; an argument is actually being presented not to Lyra at all, but directly to the real reader in this real world. It must be in this world, then, that we address the linking of Eden with the complex.

Adam and Eve (metaphors for things religious) do not exist, Pullman seems to be telling his readers. It is just that religion helps us to understand things that would be too hard to understand without it. Imaginary numbers help mathematicians and scientists understand things that are too complicated for you to even contemplate, so don't even bother, just learn how to multiply 2+3i by 7-5i; just learn who begat whom. And there actually is something just so insidiously patronizing going on here; people who believe in Adam and Eve (or that for which they are metaphor) only do so because there are things that are too hard for them (poor children) to understand without such crutches. The problem with Pullman's metaphor, however, as nice as it appears on its face, is that imaginary numbers, as I hinted above, are not imaginary; the name is a misnomer.

That a middle school mathematics teacher should have the understanding that imaginary numbers do not really exist, and pass this understanding on to his students, is not really surprising. It is certainly forgivable. Even Descartes used the term "imaginary number" dismissively. Indeed, this was the understanding of the entire mathematics community for the first two centuries after the concept was introduced by Cardano in the early 1500's in his *Ars Magna*, the same work in which he introduced negative numbers. It should be said that negative numbers were also viewed with suspicion, and so it is little wonder that their square roots were not easily accepted until the genius of Euler demonstrated their power.

Before we can get a more modern notion of what imaginary numbers are, we need to understand what is meant by *dimension*. Pullman himself dabbles with one of the stock science fiction motifs, that of parallel universes, what are often, mistakenly, called "other dimensions". He, quite correctly, does not use the expression "other dimension" to name his parallel universes, but, his multiverse is, of necessity, multi-dimensional. What does that mean?

Start with a line. Imagine that the line is a universe, and that beings live in it. These beings can move in only two directions, forward or backward. We introduce the concept of sign and use + for forward and – for backward. In this sense, beings in our universe can move in only one signed direction, that is, along the line. We call a signed direction the dimension of the universe. Our line is a one dimensional universe, since it has only one signed direction in which its inhabitants can move.

Now imagine a universe that is a plane, like the surface of a table, or like a blackboard. In the middle of the plane there is a point surrounded by the twelve hours of a clock face. It would seem that beings in this universe can move in an infinite number of directions, but, in a sense, they can move in only two. Consider the two directions given by moving from the centre of the clock to 4 and from the centre of the clock to 11.
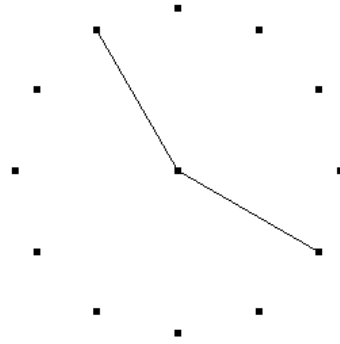
Notice, first of all, that the signed (forward/backward) direction from the centre to 4 is exactly the same signed direction as from 5 to 9, 8 to 6, 12 to 2, etc., and that the signed direction from centre to 11 corresponds with the direction from 12 to 4, etc. Imagine that the distance from the centre to the clock points is one mile. One can get from the centre to 1 by going two miles in the "11 direction", followed by about 1.7 miles in the "4 direction".
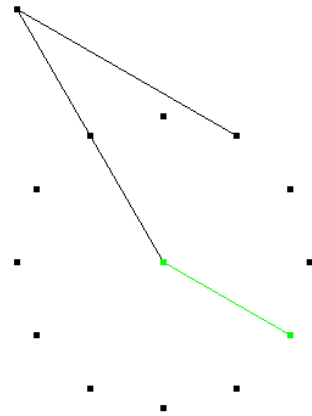
In other words, going from the centre to 1 is not really a new direction at all. It can be accomplished by using the first two directions. Indeed one can get from any point in the plane to any other point in the plane by going a certain distance in the signed "11 direction" followed by a certain distance in the signed "4 direction". If there was some universal law stating that one could move in only these two directions, beings in this universe could still move between any two points using only those two signed directions. On the other hand, if, instead of the "4 and 11 directions", our universal rule stated that we could move only in the "2 and 3 directions", we could still get anywhere in this universe. All that is needed to get anywhere in this universe is any pair of non-parallel signed directions. This is a two dimensional universe. For a universe to be two dimensional we mean that only two predetermined signed directions are ever needed to get from any place to any other place in the universe.

In a one dimensional universe, all that is needed to get anywhere is one signed direction. In a two dimensional universe, all that is needed are two signed directions. Similarly, all that is needed in a three dimensional universe is a trio of signed directions.

Now, imagine that the one dimensional universe – the line – is scratched into the two dimensional table top, which is in a three dimensional room. For a being in the line, leaving the line means going in a direction other than the one signed direction of that universe, and, what is more, going in any direction other than the One, will take it out of its universe. Going in a different direction and leaving the universe are one and the same thing. For a being from the table top universe, leaving the table top means going in a direction that cannot be achieved by some combination of the "4 and 11 directions" and, as before, going in such a direction will automatically, take the being out of its universe. Leaving the universe, no matter what dimension that universe has, means, by definition, that the universe must be embedded in some higher dimensional space, because there must be a direction – a dimension – to move in that is not one of the directions in the home universe.

Lyra leaves her universe and travels through a multiverse to other, parallel universes. She does not, as is so often said in popular Science Fiction, travel to a "different dimension". One moves *in* a direction not *to* a direction. She moves in a dimension (direction) other than the three that define space in her own universe to get to another three dimensional universe. It is as though the line on the table top had, close by, another line running in the same direction and a being from the line universe traveled in a direction that is possible on the table (but not the line) to get to the second line; or a being from the table top rose to a plane hovering just over its home universe. Pullman understands this, and uses exactly this table-top analogy, toward the end of his trilogy, to describe the move between worlds.

When negative numbers were first introduced to Western Mathematics, they were met with disbelief. How on earth, could one possess a negative quantity of something? How could a line have a negative length? We are quite familiar with this concept today, and our children are perfectly at home with the idea of a temperature being $-10\ ^0C$ that is, ten degrees *below* some standard temperature (in this case the freezing point of water). It should be noted, though, that when Cardano wrote his *Ars Magna*, such was not the normal currency. Indeed, he was using the idea purely arithmetically and symbolically.

We write quadratic equations in the form $ax^2 + bx + c = 0$ where *a, b* and *c* represent different known numbers, which might be positive or negative. However, if we do not have a concept of negative numbers, we cannot conceive of the equation $x^2 - 2x - 5 = 0$ in the generic form above; we have to write $x^2 = 2x + 5$. Cardano, by introducing the symbolic negative allows us to write all quadratic equations in the same form and realise that, for the top equation, $x = (-b \pm \sqrt{(b2 - 4ac)})/2a$. This kind of symbolic use forces the question of just what negative numbers are. However, not daunted, Cardano goes on to use methods equivalent to the Quadratic Formula above and shows that, for some equations, the unknown *x* must contain square roots of negative numbers. In other words, immediately after introducing negatives, Cardano gives us imaginaries, without ever giving us any real notion of what either mean!

Quite soon after the *Ars Magna*, people did realise that negative numbers could describe indebtedness. As a matter of fact, Shakespeare, writing towards the end of the same century used precisely this construction in *The Merchant of Venice.* We begin to see an understanding of negative numbers evolving. Indeed, the child's understanding of a temperature ten degrees *below* zero is significant. *Below* implies a direction – down. While an arithmetical understanding of negative numbers to describe indebtedness might be of interest, and certainly easy enough for middle school students to understand, it is the geometrical understanding, the notion of going backwards, that is of importance here. I have, of course, already been using this sense of the negative in describing dimension: a dimension is a signed direction, a direction in which one may move forward (+) or backward (–).

Dimension is key to an understanding of Pullman's work. If there are not higher dimensions, then it is impossible to move between universes. Indeed, only one universe exists. Moreover, when we are considering space, the universe, the multiverse, we are, of necessity, thinking geometrically. We use the concept of negative numbers not to describe being in debt, nor to describe a temperature colder than some standard, but to describe moving backwards.

Positive and negative numbers describe the two directions our one dimensional line dweller can move, forwards and backwards. Suppose that the one dimensional universe is the (extended) line that joins the centre of the clock to the 4. The being in that universe cannot conceive of the "11direction". To it, that direction is imaginary.
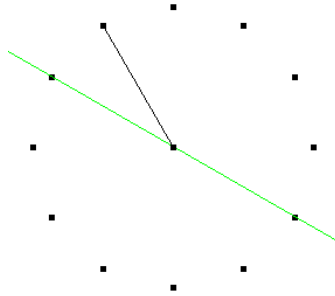
That direction exists; we can perceive it, as we can perceive the whole of the two dimensional table top, pre-cisely because we are not one dimensional beings. Yet the direction remains imaginary to the line dweller. This use of the term "imaginary" is not an equivocation, although it is a slight mathematical inaccuracy. For about two centuries now the mathematical community has worked with a geometrical understanding of imaginary and complex numbers, and would call this direction *complex*. As far as the one dimensional being in the diagram above is concerned, modern mathematicians would call the direction from the centre to 1 (or 7) the imaginary direction. We perceive the 11 direction above as not entirely imaginary since a being moving in that direction is still moving in approximately the NW "real" direction. However, to move from the centre to 1 is to move in a purely imaginary direction. It is to move completely away from the real line.

In traditional, Newtonian mechanics a moving object will continue moving at precisely the same speed in precisely the same direction unless it is acted on by some force either to speed it up, or to slow it down, or to change its direction. A force that speeds the object up without changing its direction is described as positive; a force that slows the object down without changing its direction is described as negative. We use numbers to describe how large the force is. How do we describe a force that changes the object's direction? The force clearly has some magnitude, and, just as clearly, must not be in either the positive or the negative direction, or else it would merely change the object's speed, and not redirect it. The answer is, that we choose some direction other than that in which the object is moving, give it a forward and backward(+/–) sense, and call that the *imaginary* direction. A force of, whatever size in this direction will redirect the motion of our object. Moreover, if we are just talking about movement in our table top universe, forces in these two directions (along the motion of the object and in the imaginary direction) are all that is needed to affect any change whatsoever in the motion of the object. We describe forces that are in a combination of the two directions using *complex* numbers. Of course, in our three dimensional space, we would need to describe our forces in terms of three fundamental directions.

This all seems reasonable, but the astute reader will object: but what about all this nonsense of $i^2 = -1$? There is some arithmetic here that still needs explaining. How does this strange piece of arithmetic match up with the geometry of objects moving through space? In the geometrical sense of imaginary and complex numbers, multiplication describes growth and rotation. Imagine a stationary being on the table top. Multiplying it by any positive number describes its size increasing or decreasing (depending on whether the number was greater than one or else a fraction). As with beings so with other less tangible things like movement, multiplying a force by a positive number may change the magnitude of the force, but it leave alone the direction in which the force acts. Multiplying by 1 means leaving an object or a force alone – just as multiplying by one is a neutral action in regular arithmetic. Multiplying by a negative number describes the being rotating so that it faces backwards and grows or shrinks depending on the absolute size of the number. Multiplying a force by a negative number describes a new force acting in exactly the opposite direction as the original. In particular, multiplying by – 1 describes the object facing backwards, the force pointing in the opposite direction. Notice that multiplying by – 1 twice, we reverse directions twice and are back where we started. The square of a negative number is positive, just as in our standard arithmetic.

The interesting thing is what happens when we multiply by an imaginary number. Multiplying the object or force by *i* describes leaving its size alone but rotating it through *anticlockwise*. Multiplying the object or force by *i*

again rotates through another 90° and the object is facing backwards, the force is pointing in the opposite direction. This is exactly the same as what is described by a single multiplication by – 1. Multiplying by i two times describes the same action as multiplying by – 1 once. $i^2 = -1$

Do imaginary numbers exist? Do negative numbers exist? One might just as well ask if real positive integers exist. To be sure, we all know what two dogs are, or three houses or seven students, but do 2, 3 and 7 exist? Aristotle lists quantity as the second most fundamental predicate of being, and, in this Aristotelian sense, natural numbers exist in as much as they are descriptors of the universe. Since, today, we have a much expanded sense of number, so too we expand our sense of Aristotle's second predicate. 2 exists because it describes two dogs. -10 exists because it describes a temperature 10 degrees colder than the freezing point of water, –153 exists because it describes being $153 dollars in debt, –7 exists because it describes a force of 7 units directly against the motion of some moving object, and –1 exists because it describes a facing backwards. In precisely this way, 5i exists because it describes a force of 5 units out of the line in which an object is moving and i exists because it describes a quarter turn. A complex numbers such as 3+4i exists because it describes a combined force, speeding an object up by a factor of 3 units in the direction in which it was already moving and 4 units in the imaginary direction. Such a number also describes a rotation of about 53° combined with a growth of a factor of 5, but, perhaps this is straying a little too far afield.

Pullman's analogy, then, does not quite deliver what he wants it to. Far from being non-existent or, at the very least, unprovable, imaginary numbers do exist. If we should think about Adam and Eve like an imaginary number, then we have to think about them as existing and relevant. As so often happens, a nice, seemingly wise comment about spirituality really takes us nowhere at all. It neither undermines the Christian tradition nor, to be honest, does its nullification do anything to enhance it.

We are left with one question: is Pullman's own understanding of complex numbers the middle school understanding of his former students and teaching staff? Is he honestly misapplying an analogy with things he does not really understand, or is there something happening that deserves rather less charity? Does Pullman actually understand complex numbers but, knowing what a middle school understanding is, use that understanding to make an analogy that suits his purpose? If that is the case, then he is being less than honest with his readers. Indeed, deceit is put forward as an admirable quality in the trilogy – Lyra, herself proudly bears the title "Silvertongue" for her ability to lie convincingly – and even the starting quote displays some dishonesty on Asriel's part since, as we have seen, this character does have some belief in that for which Adam and Eve are metaphors.

We have already seen that Pullman has an understanding of the dimensional implications of moving between parallel universes. He also has an admirable description of a multidimensional fractal Fortress of Heaven in the Shining Mountain of *The Amber Spyglass.* Indeed, Pullman's description of a multiverse of discrete universes splitting off from each other depending on whether events do or do not happen, a multiverse created by subatomic coin flips, is remarkably consistent with models of some modern currency. One wonders how an author is able to be so consistent with current mathematical physics, so comfortable with notions of higher dimensions, and yet not be the master of something that is mathematically considerably more basic. For that matter, how can a multiple dimensioned multiverse exist if the two dimensions of the table top described by complex numbers do not?

God, like Mr Pickwick, is "real in context". This analogy is really only useful if we are certain that the *contexts* are equivalent. It can be argued that "reality in context" is the only kind of reality that numbers have. Even the basic counting numbers from 1 to 10 exist only in context, and all numbers, even the purely "imaginary" ones, share this meta-existence. Perhaps, as Pullman says, God is like this, like Mr Pickwick, like the square root of minus 1, but the contexts, the stories, of Pickwick and i are fundamentally different from each other; i exists, as we have seen, no less than 1 exists.

It is interesting to note, however, that if we were to paraphrase the opening quote of this essay "But think of God like a real number, like 1", the Christian community, not to mention the Jewish, Sikh, Jain, Moslem and numerous others, would respond "We do."

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