

Reasoning with Paradoxes in a Community of Mathematical Inquiry: An Exploration Toward Multi-dimensional Reasoning

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Abstract

In this paper, reasoning is understood as «integrated reasoning» – that is, reasoning in a four-dimensional modality, manifested as formal, informal, interpersonal and philosophical. The first two modes are practiced in school to a greater or lesser degree, whatever the subject matter, but the last two are typically ignored by teachers except episodically. Formal reasoning is here understood as reasoning which is limited to obtaining definite results by applying explicit rules to clearly defined concepts and statements. In this sense, formal reasoning is only possible under special conditions within isolated and limited systems, such as formal logic or mathematics. But whenever we use a mathematical model to approximate, study or predict a real-life situation, we are necessarily bound to resort to the use of informal reasoning as well. Furthermore, the necessary hermeneutical process which is involved in the business of transitioning from the real world to formal systems and *visa versa* is far too complex to be reduced to formal and informal reasoning. It necessarily contains a metacognitive component, which is responsible for sustaining consistency as well as for coordinating between a real world situation and its formalized mathematical model. As such, it is a necessary component of creative thinking, problem solving, and «critical consciousness» in general. This dimension, which is referred to as philosophical reasoning by Cannon and Weinstein (1993) makes critical and evaluative thinking both possible and welcome in the mathematics classroom. It has the capacity to sustain ongoing doubt and inquiry through keeping various alternatives open. Finally, since most of the time the hermeneutical process is a social enterprise which involves reasoning with others who start from different points of view, it necessarily requires an ability for interpersonal reasoning, which is characterized by an emphasis on respecting others' opinions and arguments, careful listening, and on committing oneself to a search for meaning through the negotiation of multiple perspectives.

This paper argues that the traditional approach to teaching mathematics and reasoning in our school tends to separate processes and isolate elements for purposes of

analysis, thus rendering them static and acontextual, and thereby reducing mathematical thinking to formal reasoning within a purely formal system. All that remains in such a methodology is a monological emphasis on the product, which precludes real attention to the reasoning processes which led to it, or to questions such as «why» or «for what purpose,» or «what does it mean?» This critique implicitly suggests a re-examination of the relationship between mathematics and «the real» world that transcends the conventional framework of school mathematics in directions that connect formal and informal mathematics, and mathematical-logical reasoning with other dimensions and domains of reasoning.

Because the collective and distributed aspects of reasoning are still unaccounted for in research in mathematics education, integrated reasoning is a relatively unknown quantity, not just in the literature of the field, but in the teaching of reasoning in the classroom. There are some exceptions—when, for example, reasoning is practiced in a «community of inquiry,» which emphasizes the development of the skills and dispositions of communal dialogue and argumentation. This paper includes a short analysis of excerpts from a series of transcripts of conversations with fifth graders, which finds clear evidence of all four dimensions of integrated reasoning to be operating in the dialogue, thereby implicitly arguing for the notion that communal discussion—in this case, communal inquiry into paradoxes—tends to stimulate all of them.

Reasoning as a subset of thinking is an area of inquiry which has long been a preoccupation of cognitive psychology, human development, and pedagogy. Deductive reasoning can be defined as one or a series of cognitive moves from one state of a problem situation—represented by a set of premises—to another state in the form of a conclusion, in which the truth of the conclusion necessarily follows from the truth of the premises. And whereas reasoning is one type of thinking, deductive/formal reasoning is one type of reasoning in which the inference process—assuming that it follows a specific form—advances a conclusion whose premises and their negations are inconsistent.

I will use the term «formal reasoning» hereafter to denote reasoning which is limited to obtaining definite results by applying explicit rules to clearly defined concepts and statements, and to the demonstration of methods of proofs in formal logic and mathematics. This form of reasoning is very much in use in modern logic, where logicians work within a purely formal system, and where the sole concern is to choose the symbols and rules that are permitted within the system. Any considerations that have to do with building new statements or theorems of the system, or new rules of deduction, or changing the meaning of expressions, or the role of these expressions and rules in the elaboration of thought, are foreign. Any interpretation of the elements of the axiomatic system are left to those who will apply it and who will take the responsibility for their adequacy and meaningfulness in the real

world. In this sense, formal reasoning is only possible under special conditions within isolated and limited systems, such as formal logic or mathematics.

However, we use mathematics and formal logic for the very reason that they both try to model the «real world.» Hence, whenever we use a mathematical model to approximate, study or predict a further development of a situation, we are necessarily bound to simplify it. We resort to logical reasoning within the model that we use, since the formalization of the situation requires simplification and formalization of the thought. However a formalized model cannot uniquely determine its own reference—a descriptive corollary of the Lowenheim-Skolem theorem, which holds that an axiomatic set cannot uniquely identify a set of objects (Banach, 2005). It has implications for the establishment of reference between a formal model and the «intended interpretation» of this model—i.e., the context that is intended to be approximated by the model. The Lowenheim-Skolem theorem supports the indeterminacy of such a unique reference relation—that is, a formal model does not uniquely define an intended context (Weinstein, 2002). The final determination of this context has to be made on the basis of intuition, individual judgment or social agreement—that is, as an interpretation. This hermeneutical process is thus sustained by judgmental and not by propositional thinking (Rosen, 1987). What was said above necessitates the conclusion that formal reasoning provides only an underlying structure, and needs a complement for the more complex hermeneutical business which is multidimensional or what is here called integrated reasoning.

Integrated reasoning deals with reducing and applying data, i.e. it is concerned with transitioning from the real world to formal systems and *visa versa*. It deals with transposing concepts from one world to another, and operationalizing these concepts as terms, which requires working with definitions, interpretations, and real or apparent irrelevancies, and making the necessary revisions in order to maintain consistency. Integrated reasoning necessarily contains a metacognitive component, which is responsible for sustaining consistency as well as for coordinating between a real world situation and its formalized mathematical model. As such, it is a necessary component of creative thinking, problem solving, and «critical consciousness» in general.

One way to characterize integrated reasoning is as a form which operates through what Wittgenstein calls «doubt and enquiry» (1969, p. 151). Another descriptor is suggested by Cannon's and Weinstein's (1993) classification of four dimensions of reasoning—formal, informal, interpersonal and philosophical—all of which, as they point out, are not necessarily to be thought of as progressing sequentially, or manifesting in all four dimensions at any one time. Some of these dimensions might stay underdeveloped if not cared for and encouraged. The first two modes of reasoning—formal and informal—are practiced in school to a greater or lesser degree, whatever the subject matter, but the last two are typically ignored by teachers except episodically.

The third mode—interpersonal reasoning—involves reasoning with others, starting from different points of view and moving towards common agreement through argumentation. It is characterized by an emphasis on respecting others' opinions and arguments, careful listening, and on committing oneself to a search for meaning through the negotiation of multiple perspectives. This mode of reasoning is

best learned in a community of inquiry, where one can develop a dedication to practicing reasoning through dialogue. The fourth dimension—philosophical reasoning—is, according to Cannon and Weinstein, better defined as reflection on one’s own reasoning, or what was previously referred to as the ability to do meta-analysis. It is this dimension which makes critical and evaluative thinking both possible and welcome in the mathematics classroom. It has the capacity to sustain ongoing doubt and inquiry through keeping various alternatives open. Integrated reasoning encompasses all the four dimensions described above, which suggests that, in the practice of teaching it should be at the very heart of what education for good reasoning is all about.

The pragmatic approach to reasoning taken by scholars in the field in the 1960’s redefined reasoning, and included informal reasoning as one of its legitimate forms. This new approach also gave rise to the new field of «critical thinking» and encouraged the old sub-discipline of argumentation (with rhetoric as its antecedent) to flourish. Nowadays argument is not only recognized as inherently social, but also as a form of communication (e.g. Walton, 1998; van Eemeren & Grootendorst, 1992). The opening of a formerly highly restricted area of reasoning led to the integration of argumentation with communication theory, and helped move it away from a rigid formalism (van Eemeren & Grootendorst, 1992). In fact, recent trends in argumentation theory acknowledge the necessity of both formal reasoning and a hermeneutical process, which implies dialogue, and authors like Douglas Walton, Frans van Eemeren, Rob Grootendorst , and Francisca Henkemans have introduced even newer approaches to argumentation understood as a form of dialogue (van Eemeren, Grootendorst & Henkemans, 1996; van Eemeren & Grootendorst, 1992; Walton, 1996; Walton, 1998). All of these new approaches and models are context sensitive, and depend for their meaningfulness on the purpose of the critical discussion to which they are applied.

Although this loosening of the rigid association between reasoning and formal logic has allowed some other dimensions of reasoning into the inquiry, argumentation is in fact still mostly viewed as an individual rather than a social activity, and analyzed mostly from a Piagetian perspective (e.g. Willard, 1989). Because the collective and distributed dimension is still unaccounted for, interpersonal reasoning is a relatively unknown quantity, not just in the literature of the field, but in the teaching of reasoning in the classroom. There are some exceptions when, for example, reasoning is practiced in «communities of inquiry,» which emphasize the development of the skills and dispositions of communal dialogue and argumentation. In fact the primary focus of a community of inquiry—as its name implies—is the fourth aspect of reasoning—meta-reasoning. Although invigorated by theories of metacognition that have inspired research in mathematical reasoning over the last fifteen years (e.g. Schoenfeld, 1985, 1992; Garofalo & Lester, 1985; Carpenter & Fennema, 1992, 1996), the latter does not seem to have drawn much attention among dialogue and argumentation theorists, perhaps on the presumption that reasoning capacity tends to appear as an individual strength or weakness in the reasoner, rather than a skill or disposition which can be developed through a communal, distributive process. The argument advanced here holds that community of inquiry pedagogical model creates conditions for multidimensional, or integrated reasoning.

The short analysis which follows, based on excerpts from a series of transcripts of conversations with fifth graders— will explore all four dimensions, thus implicitly arguing for the notion that communal

discussions and, as in this case, communal inquiry into paradoxes, tend to stimulate all of them, and therefore can be characterized as eminently conducive to what I have called integrated reasoning. The discussions in question took place once a week, in a public school in northern New Jersey. During each session the students were presented with a paradoxical problem which they discussed and tried to resolve in a community of inquiry format. The community of inquiry discursive model demanded careful listening , and required them to make arguments and offer grounds for them, to respect each other's opinions and arguments, to reflect on their own or someone else's statements, to search for alternatives for premises and arguments already presented, and to consider different points of view in arriving at plausible solutions. The role of the facilitator in the discussions was directive rather than assertive, modelling a discursive medium which allows for the distributed participation of all students in the dialogue.

The community of inquiry format emphasizes the interpersonal aspect of the reasoning process, and thus provides for a discursive structure which encourages the other forms of reasoning identified by Cannon and Weinstein—formal, informal, and philosophical. In these sessions, formal reasoning was present for the most part as reasoning about the relationships and conditions from which students' conclusions were drawn. Informal reasoning was evident in the students' interpretations of the problems, their explorations of ambiguities, in the addition of unstated premises, and in their evaluation of whether the «model» which they considered to represent «the real situation» made sense. Finally the philosophical facet of the reasoning was manifested in these dialogues in the critical exploration, clarification and analysis of concepts nested in conflicting relationship, and in the examination of criteria for evaluation based on the analysis of hidden assumptions. The following selections from over one hundred pages of transcripts are offered in order to provide examples of all four dimensions of reasoning, and to make the case that, when allowed and encouraged to do so, what I have called integrated reasoning is a natural and powerful form of collaborative mathematical inquiry.

Episode 1

Facilitator: Here's the problem for our discussion today: Imagine a village far away in Greenland. There is a man there who is a barber. This barber shaves all and only those men in the village who don't shave themselves. Does the barber shave himself?

The discussion started somewhat chaotically. Darlene, Sally, Jimmy and Bill, not quite grasping all of the conditions, reiterated the problem, almost as if thinking aloud in an attempt to stay aware of their own inferential process.

Darlene: He has to shave himself, because he shaves all the men in the village who don't shave themselves. And if he is the one who doesn't shave himself then he has to shave himself.

Sally: The question says...that he shaves only those who don't shave. Maybe he does. Maybe he shaves himself. [She doesn't seem so realize that the conditions of the problem don't give the option «maybe»]

Jimmy: He is his own barber. He shaves other people, but maybe he needs someone else to shave him. [Jimmy is trying to evaluate two different ideas, and is unable to decide which one is better]

Bud: If he shaves the men who don't shave themselves, then if he doesn't shave himself, then he has to shave himself. [Ben is the first to realize the absurdity of the conclusion]

Naomi: I think this problem is like impossible and really hard, because he shaves only people who don't shave themselves, so he can't shave himself ... And he can't shave himself, because he shaves *all* the men in the village who don't....which would mean that he has to shave himself.

This circle of logical reasoning led to deducing the absurdity of claiming that the barber shaves himself if and only if he does not. Confronted with a *reductio ad absurdum*, Naomi concluded what any logician would conclude—that «It's an impossible situation»— and the group agreed. But the group continued by challenging this model, which turned out not to be adequate to a real life situation. They attempted to «fix» the model through redefining, reinterpreting, going back and forth between the mathematical model and «real life»—a spiral process of analysis and meta-analysis, or reasoning in an integrated way. A few ideas emerged as to how to fix the problem, such as, the barber has a beard [Naomi]; the barber is a not a *man* [Victor: «.... It never said he is a *man*. They never said *him*.»]; or that he travels to another barber outside the village to get shaved. Sally even suggested that the barber plays the role of a barber only when he is at work and «...there he shaves [all] those who don't shave themselves,» while at home he is not a barber, thus trying to «split» the barber's personality in an attempt to avoid the contradiction.

Victor: But it never said that he's a man. They never said «him.» Read it again.

[The problem is read again]

Voices: It can't be a woman. Oh, you're prejudiced.

Facilitator: If the barber is a woman then the question won't be relevant. Let's assume that the barber is a man.

Victor: The only answer then is that the person travels to another barber out of the village to get shaved. It's outside the problem, but I can't find any other answer.

Al: Chas said it, because he's the one who shaves all that don't.

Sally: One minute, it's like....When he's at home he shaves himself, but he isn't a barber there, but when he gets into the barber shop, he's a barber and there he shaves those who don't shave themselves.

Bill: It's like a teacher. Teachers usually go to another town or county to teach....most of the time.

Rush: O.K. Yeah. Let's say you're Ms. H. [the classroom teacher] for instance. Let's say this is a school to teach other adults and say, she's an adult herself. She would go to another place to be taught. So, the barber cuts people....shaves people, but he goes to another barber's shop to get shaved.

Interestingly enough, these students' attempts to avoid contradiction echo those well-known efforts to defeat the logic of paradoxes by well-known mathematicians. Eventually the question of how tight or loose our interpretations of the problem were, and what could be considered acceptable in that

regard, emerged. As a result of this discussion, the first two ideas were discarded, since they were making the initial question appear irrelevant. The third idea was rejected on two grounds: Jimmy's argument that «. . . the actual story never said that he goes to *another* barber shop»; and that by making interpretations we were «adding stuff.»

Jimmy: I'm sorry, but the actual story never said that he goes to *another* barber shop. It never says that, a.... there is another barber *but* him. It never says *anything* that we're practically saying except for he can only shave people who don't shave themselves and it never says that he can't shave himself or he can. He might have problems like he can't reach his hand to his face, like, man, *we could put so many loopholes in this thing...*'cause, like all we're doing is *adding stuff* to the story.

This intervention in fact forced the participants to reflect on what we were doing: were we inserting new data into the schema or uncovering hidden data (premises)? Naomi added another argument against the third idea, emphasizing again the quantifiers «all and only,» and the logical necessity they impose. And the final conclusion came as an agreement among the whole group: «There isn't such a barber.»

Voices: There is no solution. There isn't such a barber.

Victor: He goes out of the village, shaves himself and goes back to the village. Because the conditions are only in the village....

Rush: First of all, don't you realize what they're trying to do? They pick a never ending question to see how bright and creative we can think of ideas. This question is never going to end. They pick this question on purpose, so that nobody can stop with the right answer. This is a non-answerable question. It's never going to end. Because we don't have enough proof to answer one. This is the great philosophy.

The discussion then switched to a metadiscussion about what made this problem unsolvable. Rush thought that there was not «enough information.» Victor suggested that the problem is «too much information, not too little.» The group seemed to agree that the restrictions imposed by the quantifiers «all and only» were too strong to make the problem workable.

Sally: There is no answer.

Sandra: But, what makes this question unanswerable? People were saying that this question goes in a circle?

Rush: This is another question that is unanswerable. I don't know what part of the question....makes it...like maybe that there is not enough information. But there is something about that question. Now that's going to be a tough question. I'm not sure what exactly about this question makes it unanswerable.

Facilitator: I'd like somebody to summarize. Bill?

Bill: Well, we kind of came to...it's a never ending question, because he [the barber] can't be in two different groups at the same time. And then what the other question was asking...why and what makes it like that. But that's like first asking: «What is the meaning of the universe?» and then «Why can't you figure it out?»

Rush: It's a never ending question, because it entails another question.

Victor: I think that what the problem is that we have *too much* information, not too little. Because if we didn't know that he shaves *all and only* people in the village [that don't shave themselves], we could just say «O.K. He shaves himself.» If he was a just barber, that's all we knew, then this barber could shave himself. If it doesn't say «all and only» people who don't shave themselves, then the answer is that there is too much information which *restricts us* to this conclusion. Another question is, what information do we need to find that the barber shaves himself?

Facilitator: That would be a question for a whole other session.

Sally: That might be another never ending question.

Here we see that besides the instances of formal reasoning, there is an abundance of informal reasoning going on as well—for example all the evaluations, according to criteria of reasonableness, of what is and is not within the boundaries of the problem; what is relevant and what is not; evaluations of coherence, analogical reasoning, interpretations and explorations of meaning. There is evidence of interpersonal reasoning in the fact that members of the group are engaging with each other in rational discussion, reflecting critically on their own arguments, or responding thoughtfully to the arguments of others, giving and receiving constructive points in perfecting a proposition, and working towards resolving different points of view. Finally, the philosophical dimension is manifested in this episode through the group's critical exploration, not only of the hidden assumptions about the barber, but also through a meta-analysis of the process of interpretation of the problem and of what makes the problem paradoxical.

The next episode offered here followed one week after the presentation of the Barber problem, and further exemplifies the use of integrated reasoning in pursuing the solution of paradoxes.

Episode 2

Facilitator: Today, I'm offering you this sign to read [she holds up a sheet of paper, on which is written in large letters, «Do not read this sign»]. What if I put up such a sign? Do you see a problem here? Let's comment on it.

It took the group some time to figure out what the problem with this sign was. Al observed that it seemed meaningless in terms of use. «What is the point of having this sign that says 'Do not read it'?» Naomi found another problem—that «in order to find out that you aren't supposed to read it you have

to read it.» Then Samantha helped the whole group clarify the issue by making an analogy: «It's like sending an invitation to a party saying 'Don't come'.» Sandra's call for a comparison between a sign which reads «Do read this sign» and another that reads «Do not read this sign» also helped the group decide that the first is only meaningless but not problematic, and that the second contains a hidden problem. Now it became easy to see that actually the sign offers two contradictory messages: it is offering to be read and yet it is saying «Do not read this sign.»

To assert a proposition and its negation within one and the same system reveals a contradiction within that system, and thereby renders it inconsistent and unusable. Avoiding inconsistency and absurdity requires either abandoning certain elements of the system or augmenting it in some way. So it seemed logical that the discussion moved next to an attempt to restore consistency. Several ideas were offered in hopes of avoiding inconsistency by augmenting the system. For example: «This is a kind of cut off sign and the rest is covered up. It might have been «Do not read this sign ...on Wednesday»[Chad]. «They could be talking about a sign next to it, if it's really old. So, they could be saying «Don't read this sign» about the other sign» [Naomi]. The students intuited that it was the self-referential quality of the sentence that brought on the paradox. Interestingly enough, the group reached an agreement that if it refers to another sign or message in the immediate environment, the sign seemed perfectly reasonable. But it was also recognized that this augmentation in fact played around the boundaries of the problem. The discussion later transitioned to a metadiscussion on the difference between the message in the given sign and the negation of this message, and ended in an exploration of the possibility of accepting the contradiction—a solution which, in fact, dialectical logic suggests.

Conclusion

The traditional approach to teaching mathematics and reasoning tends to separate processes and isolate elements for purposes of analysis, thus rendering them static, acontextual, and separated from potentially conflicting tendencies within the system they represent. By abstracting terms from their relationships, it neglects the multiplicity of those relationships, the opposing tendencies within and among them, and the inevitable complexity that implies.

The same reductionism still rules school mathematics, keeping it strictly isolated, shrinking mathematical thinking to the realm of the calculative, and suppressing its connections with other dimensions of human thought and experience. The result is a school discipline stripped of almost all connection with the world, in spite of the fact that mathematics is held in such high esteem just because of the extraordinary technical success which Western cultures have experienced by mapping mathematical models onto real world situations (Schoenfeld, 1991; Schoenfeld, 1988).

As a result, mathematical thinking in educational contexts is reduced to logical reasoning within a purely formal system. As a further result, mathematical-logical reasoning is deprived of proof demonstrations, and all that remains is a monological emphasis on the product, apart from real attention to the reasoning processes which led to it, or to questions such as «why» or «for what purpose,» or «what

does it mean?» The result is a form of school mathematics that is completely enclosed in its instrumental applications, and which encourages teaching a form of logical-mathematical reasoning which is in fact a form of pseudo-logical reasoning (Dreyfus, 1999).

The data offered here implicitly suggest a re-examination of the relationship between mathematics and «the real» world which transcends the conventional framework of school mathematics in directions that connect formal and informal mathematics, and mathematical-logical reasoning with other dimensions and domains of reasoning. Such a re-examination does not seem necessary apart from the recognition that formal reasoning is a necessary but not a sufficient dimension of reasoning as such; and such a recognition requires shifting the focus from a purely formal to an integrated mode of reasoning that can encompass the rich multidimensionality of the transitional domain between mathematical models depicted by formal systems and the real world. Similar proposals for a shift from a formal to a dialectical mode of reasoning have already been made in other areas such as moral reasoning in adolescents, and in interpreting life-span development in general (Gilligan, 1979; Riegel, 1973). In a similar fashion, this project suggests an expansion of the commonly accepted notions of reasoning in schools to approximate an integrated model. This in turn suggests a change in our understanding of the goals of reasoning from an objectivist and instrumentalist emphasis to an understanding of reasoning—including formal reasoning—as a tool for the pursuit of self-actualization (Labouvie-Vief, 1980). This shifting of the focus from ends to means necessarily entails the possibility of dialogue between the mathematical and the philosophical, and promises an imaginative exploration of multiple territories, in an interplay of reason and creativity, the concrete and the abstract, theoretical and personal experience, reasoning and interpretation. Paradox may be one starting point for this exploration.

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