Philosophy of Mathematics in the Classroom

ASPECTS OF A TRI-NATIONAL STUDY

The following papers were presented at a panel of the Interamerican Congress of Philosophy in Puebla, Mexico, in August 1999. The panel was convened by Dr Maria Teresa de la Garza of the Universidad Ibero Americana, who invited Dr Marie-France Daniel of the University de Montreal and Dr Christina Slade of the University of Canberra to discuss the teaching of Philosophy for Children adapted for mathematics. All three are participants in a Canadian government funded research project led by Dr Daniel, in which a program developed by Dr Daniel and her Quebecois colleagues, Dr LaFortune, Dr Pallascio and Dr Sykes, designed to teach philosophy of mathematics to childen is being tested in the original French, and in translation in English and Spanish. The round table discussion in Puebla was designed to introduce the program, and defend its place in the curriculum.

The panel begins with a broad overview of the aims of the philosophy for children program, and the importance of critical and creative thinking in the classroom (Dr De la Garza). The focus then narrows to the role of *philosophy of mathematics in the classroom (Dr Slade) and finally turns to details of the project (Dr Daniel).*

These papers argue that, in spite of the prejudices of mathematics teachers, philosophy of mathematics is precisely what is needed to create a better learning environment in mathematics. The argument relies on a particular style of the teaching of *philosophy of mathematics, which is dialogue based and derived from the Philosophy for Children model introduced by Matthew Lipman.*

Philosophy for Children

Maria Teresa De la Garza

Very few persons will disagree if we say that education is a global process dealing not only with cognitive skills development, but also with the formation of habits that are necessary for a fulfilling life within society. The expectations of parents, teachers and society is therefore that the educational system should provide a more dynamic, critical and creative education that takes into account the value dimension of the student, both moral and aesthetic.

Philosophy for Children emerged at the end of the sixties as a response to that demand. Matthew Lipman (cf 1988, 1991) mantains that philosophy is not only an «academic» discipline reserved for specialists. The process of philosophizing is paradigmatic for dialogical thinking, and it should be available to children at school from the very beginning. It is evident that it is not a matter of teaching children philosophy as if they were students in universities. What we are interested in is in the process of philosophy itself. Lipman reminds us of Socrates, who models philosophy as a praxis, as a way of living that we all could adopt. We have several reasons to teach philosophy socratically, in order to make philosophy available for children and adolescents. I will mention three: first, because we recognize the right that children have to the achievments of human culture; second, because philosophy has a role no other discipline can play so well in the integral development of human beings; and finally, because contemporary societies face the urgent need to educate citizens for a democratic life, and the best way to achieve that is through the practice of dialogue guided by the process of argumentation and through the practice of solidarity in the classrooms.

Philosophy is a discipline that takes into consideration alternative ways of acting, thinking or creating. To discover these alternatives, philosophers examine presuppositions, question what is taken for granted and speculate imaginatively, taking into account broader frames of reference. That is why philosophy brings to education critical and creative thinking. In Philosophy for Children the teacher and the students share the reading of a «philosophical novel» whose characters are members of a class that, gradually, form a community of inquiry by discovering philosophical concepts and reflecting cooperatively about them. This «community» is the model for the members of the real community in the classroom. The aim is for the members of the classroom community to use the tools and methods of philosophical inquiry on an open and problematic concept, therefore perfecting their skills such as: clarifying the problem, detecting inconsistencies, looking for evidences, obtaining valid conclusions, building hypotheses, using criteria to solve problematic issue.

A Community of Inquiry is the educational environment of Philosophy for Children. Such a community cultivates skills that can be classified in four groups: reasoning, inquiry, concept formation, and translation, all of them conducive to good judgement. Students learn to object to weak reasoning, to build solid arguments, to accept responsibility for what they say, to respect different perspectives, to practice self correction and to make good judgements in the context of dialogue.

But not only cognitive skills are developed. We can detect also attitudes that show value development, such as: listening carefully to what the others say, encouraging others to express their opinions, taking turns to participate in the dialogue and taking care of the tools of inquiry themselves at the same time, taking care of each member of the group and of the group as a whole, appreciating the beauty of the world and of hi human culture. We move in a world of values about which it is necessary to reflect and to decide.

Moral education is a central part of the educational process and since morality is not only a question of knowing rules and applying them, we need to be prepared to make decisions in concrete situations. In order to do so, it is necessary to develop moral thinking in its practical dimension that includes the development of sensitivity towards the feelings and needs of the others around us. There are three indispensable elements in moral education: an element of reasoning, another of character construction, and one of sensibility. All these elements can be developed in the context of a philosophical community of inquiry in the classroom.

Inquiry is a self correction practice, generally communitarian because it rests in language, scientific methodology and symbolic systems. On the other hand, a community is held together by praxis and by common goals and traditions. Not every community is a community of inquiry, because praxis can be, and usually is, traditonal and static. But in a community of inquiry, praxis is inquiry itself.

Inquiry in a philosophical community of inquiry has the following characteristics: it is a process conducive to a judgement. This process follows a direction that is the direction of argumentation. This process involves language in the form of dialogue

Dialogue in a community of inquiry requires certain central attitudes based on ethical principles: respect for others as persons, as sources of meaning and, respect for the process of inquiry that is based in a comittment with the search for meaning. But these ethical considerations are not sufficient to define the character of dialogue. It should be disciplined by logic.

Participants in a community of inquiry should engage in reasoning in order to follow the argument; therefore, the movements of dialogue are logical moves. As the inquiry progresses, each logical move requires another one; the discovery of new data points the way towards new perspectives; a statement forces the members of the community to look for the reasons that support it; an inference moves them to explore hidden presuppositions; a distinction leads them to seek for criteria. When the members reach agreement, the sense of direction of the community is clarified and the inquiry moves on, revitalized. These agreements are always provisional. Kant mentions these kind of provisional judgements. For him, they are those judgements used in an inquiry process, those which we know should be revised, but that we use to impulse the process of inquiry.

Logic is the spinal cord of the process in a community of inquiry. Participants think together in philosophical dialogue, developing at the same time a strong critical thinking, intellectual flexibility, moving from a narrow frame of reference towards broader ones, thus engaging in creative thinking. The participants in a community of inquiry learn to think well, to think for themselves and to think in an exploratory and original fashion.

If we think that education has to do with the developing of critical and creative thinking, we should look into Lipman's proposal which follows the model of a philosophical community of inquiry. According to Lipman, the philosophical community of inquiry fosters the development of higher order thinking, that is, the merging of critical and creative thinking. This model is based on the following considerations:

- *Higher order thinking takes place under two regulative ideas: truth and meaning.*
- *Higher order thinking combines critical and creative thinking.*

- *Higher order thinking takes into account, not only the content, but also the process of inquiry itself. It is conscious of its own presuppositions and implications.*
- *Critical thinking contains an element of creative thinking.*
- *Creative thinking contains an element of critical thinking.*

The community of inquiry based on philosophical dialogue is a context that facilitates the development of higher order thinking. Algorithms are cognitve tools that reduce the need of creative judgements in critical thinking. Heuristic perspectives tend to diminish the need of critical judgements in creative thinking. Lipman (1991) argues that the development of thinking implies to think better in a language. For him, the link betwen thinking and language is unbreakable. Children learn to think when they learn to talk. Reasoning is the aspect of thinking that can be formulated in a discourse, that can be evaluated trough criteria that helps us to distinguish between valid and invalid thinking, and finally, that can be practiced in the context of dialogue.

A community of inquiry is oriented towards the development in the participants of the capacity to make good judgments. Good judgements are the product of higher order thinking and the capacity to make them is related to the old concept of wisdom. That is, wise persons are persons who are capable of making good judgements and good judgements are the result of an adequate process.

Lipman (1991, 193f) offers the following definitions of critical and creative thinking:

Critical thinking is the thinking process that facilitates good judgement because:

- *it is guided by criteria*
- *it is self corrective*
- *it is sensitive to context*

Thus, the same elements are present in both critical and creative thinking, but their priorities are different. Both critical and creative thinking are inquiry processes, but while critical thinking is focused on criteria and concepts, creative thinking is focused on values and meanings.

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Why teach philosophy of mathematics to children?

Christina Slade

Our view of mathematics suffers from two contradictory stereotypes. On the one hand, we have an image of Einstein, tearing out his hair, thinking marvellously creative and impossibly abstruse mathematical thoughts. On the other hand we have the image of the dull student on a hot summer afternoon - or a cold winter's evening - filling the mathematics work book with tedious and repetitive exercises, algorithms which require no imagination at all.

The two images offer contrasting views of mathematics as on the one hand, creative, exciting and, by the way, almost always done by males opposed to the view of the mathematics classroom - dull, imitative, and in the worse sense of the word, critical - i.e., non creative. Both images are of course limited and only partially true. However, they have an undeniable currency. Put very roughly, the argument of this paper is that we can move the teaching of mathematics from the latter to the former style by introducing philosophy of mathematics in the classroom.

This is not - at all - to say that we can revolutionise maths teaching and turn it from the hard grind we all know and love into an encounter group. As one who has actually taught in high school mathematics classroom, I am aware of the enormous effort and intelligence many teachers put in to enlivening the mathematics curriculum, and the miracles they achieve in inspiring their students. I do think that the mathematics curriculum has often been limited, and that philosophy of mathematics can usefully be added, even if whole sections of algorithmic skills had to be omitted. Nor do I think that the philosophy of mathematics is easy. I think it is different from traditional mathematics pedagogy and is, I will argue, worth while.

The general lines of the argument and the programme for teaching philosophy of mathematics in the classroom will be presented by Marie-France Daniel. The program she and others have developed in Philosophy of Mathematics has a strong focus on enabling students to understand philosophical issues, which subtend the classroom exercises. She, together with Louise Lafortune, has also insisted on a strong gender element in the program. I wish to concentrate on the notion of creative thinking within mathematics, and the question of what it would be to teach mathematical skills with a view to creativity.

I begin with a preamble on the notion of creativity, and more specifically, creativity in mathematics. I then consider a debate in philosophy of mathematics, that relating to the nature of infinity as the sort of debate which might foster creative thinking in mathematics, and discuss an early experiment in using logic to introduce algebra. I conclude with some suggestions about how we might evaluate creative thinking in mathematics.

CREATIVITY IN MATHEMATICS

Defining creativity is not easy, for any field of endeavour. Margaret Boden (1994,76-77) draws a distinction between H creativity (historical, revolutionary for the whole world) and P creativity (personnel, psychological). A theory of H creativity tells us why Einstein's work is important, and has survived when others' work did not. A theory of P creativity, for the individual is a much broader theory, about how we all create new ideas, even if not ideas revolutionary for the entire world. All H creativity is P creative, but not vice versa. In terms of teaching, it is only P creativity that can be taught, and no degree of P creativity can guarantee that an idea will turn out to be historically important.

The first and simplest reaction to the contrast I drew above between dull application of algorithms and the excitement of revolutionary theory is to say that the creativity of revolutionary mathematics, (H creativity) is only available at the highest level of mathematical thought, after years of dull practice of algorithms. This point is well taken. But at each stage of learning mathematics, each practitiioner can be P creative, can learn and apply material that is new for them, in a creative way. The experience may be all too rare in conventional mathematics classes, but it certainly occurs. Of course, algorithms have to be learnt, but they are not inconsistent with excitement. Or so I argue.

In talking of mathematics, there is little force to the romantic tendency to think of creativity as somehow illogical, non analytic, or even inexplicable in rational terms. We find this tendency in literature, but also in accounts of science (Kuhn, 1971). The product of genuinely creative thought is envisaged as discontinuous with earlier ideas, and strictly incompatible with them. This model has little attraction when talking of mathematics. If a mathematical idea, however creative, works, then it has to be justified logically, in analytic terms. As I have argued elsewhere (1999 forthcoming), creative thought is not different in kind from critical thought, but is a difference in degree. Mathematical creative thought uses the same resources, the same thinking skills as critical mathematical thought but it uses them differently, no longer following the alogorithm uncritically, but developing alternatives.

Boden's (1994, 74ff) account is useful. She talks of Artifical Intelligence models of creativity, in which we can contrast exploring the conceptual space (defined computationally), with a process of questioning or extending that space. In order to extend the space, the rules and processes of the space must be fully understood. However, in extending the space very often small alterations of rules may change the output quite radically. Her examples include modelling of genetic algorithms in which the effect of minuscule changes over generations is quite revolutionary.

The other useful model for revolutionary mathematical thought is what I have earlier called a recombinant model, which Koestler, 1974, describes as the application of analogy across disciplines. In mathematics the process of cross fertilising by useing methods from one sub discipline in another has been immensely fruitful and powerful - the application of algebraic methods to geometry, for instance. In both these cases, there should be no suggestion that the creative idea is somehow discontinuous in method or illogical. There is of course no algorithm for generating reliably H creative ideas - but that is as we should expect. What counts as revolutionary in a society is a historical and social affair, as authors in ed Boden (1994) attest. There are, however, methods for suggesting P creative strategies in mathematics as in artificial intelligence.

As we might expect, they include the strategy of becoming fully conversant with a field, exploring its boundaries, as Boden puts it. How does this apply when we think of classroom practice ? Does the application of a thousand examples enable a student to explore the boundaries of a mathematical area of thought ? Surely not. Rather, we would look for a process of exploring the application of a mathematical rule, then drawing back and questioning the application of the rule.

The word questioning is important because it implies debate, interaction, not following an algorithm. Of course, best practice in mathematics teaching has always done just that - explored the meaning of the rules while applying them. But the method of testing in mathematics has militated against such practaice. If the way to do well in a math test is to do the unique algorithm appropriate for the problem, then the game will be to find the right algorithm, not to test or extend the application of the rules.

But there are alternative ways of doing mathematics. They would involve allowing kids literally to ask questions about the application of the algorithms, to ask why they apply. It would involve asking kids to use their imagination about the conceptual spaces of mathematics. And it might yield some of the excitement which is a characteristic of creativity - and even of P creativity.

The suggestion here is that debate about the philosophical issues inherent in mathematical practices can generate just the sort of creative excitement which is so signally lacking in the mathematics classroom. The notion of debate is that developed by Matthew Lipman in the Philosophy for Children program, in which children are asked to talk together about things that they wonder about, according to criteria of rationality. Its application to the teaching of mathematics has been the work of Marie France Daniel and her team, (1995,1996 1996a) with which I am lucky to be associated.

TWO EXAMPLES

What does it mean to introduce philosophical questions in a mathematics class? I give two examples, the first drawing from and extending the work of Daniel et al, the second on my own work some years ago in Australia in teaching algebra through the prior introduction of informal and then limited formal logic.

Infinity

Philosophical debates about the nature of infinity are related to attitudes to mathematical reality. For a platonist in mathematics, mathematical reality exits independently of our perception of that reality. The consequence of that view is that infinity also exists independently of our perception. How infinity is conceived is then a further question. For platonists, the infinite - and thus the transfinite - domains exist as completed totalities in the same way as any finite set exists.

For constructivists in mathematics, the notion of an independent domain of mathematics is incorrect. Mathematical knowledge is a construct, and mathematical objects are constructed by us, the mathematicians. Of course, for constructivists, as for platonists, the rules cannot be altered at will. In Wittgenstein's image it is as we are creating a railway line across a terrain - once laid, there is no choice but for the train to follow. In such a view, there is a fundamental difference between the finite and infinite totalities, in that the latter is defined by the process of construction, by the possibility at any stopping point of being to add more. The notion of infinity is then of the emerging totality. For some constructivists, this outlaws altogether the possibility of a transfinite domain - we can only conceive of infinity as under construction.

When we first teach young children about infinity, we begin with a constructivist model - infinity is when you can keep on going and whatever number you get to, you can add one more. But we quickly slip into platonist modes of talk in the primary school and not until calculus is introduced in high school do we return to notions of limits. Of course, we cannot introduce calculus in grade school. But what Daniel et al do is introduce a philosophical set of questions about infinity (1996, 141-2), in which kids are asked what it means to count to infinity, whether they can imagine infinity, whether because they cannot imagine a thing it would follow that it would not exist. Other questions are more specifically mathematical - how can a computer contain an infinite number of possibilities?, and even harder, is the infinity of whole numbers the same as the infinity of fractions?

There is nothing easy about these questions. Finding a solution is almost unthinkable, but the process of debating what counts as an appropriate answer is in itself training in seeing the limits of mathematical rules. The talk about mathematics is also fun; for kids to have ideas of their own about infinity is or can be P creative. Certainly my experience is that children talking about philosophical issues in mathematics are engaged and excited as if they feel creative.

Logic and Algebra

My second example is from earlier work of my own, before the philosophy of mathematics project and working with girls at high school level. In Slade 1989, 1 argued that girls who were relatively weak in numerical skills could - and should - be introduced to algebra through a program of logic. I introduced some Aristotelian logic and limited propositional and predicate calculus, including the functional notation. Both the use of variables, and the use of abstraction in proofs of validity were introduced without any reference at all to numbers. Each of the girls in the group of some fifteen 13-14 year olds was able to use the variables in analysis of simple argument. The class was engaged and talkative, and certainly appeared to enjoy the sessions much more than they had the remedial math classes, where they generally were obliged to repeat very concrete tasks, such as cutting up a pizza to practice fractions.

My grounds were twofold: first, there was a considerable body of evidence on the relatively high linguistic abilities of girls, compared with their numeric skills, so that it was natural to introduce abstraction and algebraic operations through language. Secondly, I was convinced that the source of difficulties with mathematics were misidentified. Often girls who struggle with algebra are capable of high levels of abstract thinking and indeed would be quite capable of the symbol manipulation required if only if were couched differently. Stereotypes of maths as a male subject; the textbooks heavily laden with gender biased examples (how many circuits of a lawn mower would this irregularly shaped lawn require?) had, I suggested, driven girls who, properly trained, might be excellent mathematicians, or logicians, away from higher level mathematics.

The evidence was gathered over a decade ago, during which time there has been a concerted effort to develop the capacities of girls in higher level mathematics. There is much new evidence. The point I wish to insist on, however, still holds. The atmosphere in what was in effect a remedial maths classroom had been one of resentment. The opportunity to talk about the role of abstraction, to learn the notions of function and then discuss it, led to a real sense of excitement. We discussed the vexed question of whether numbers existed before there were people with animation and perspicuity. Among the group were convinced platonists («of course there were numbers - there were 7 trees, even if no one said so») and rabid constructivists, glad to discover that the position they adopted had respectable philosophical roots.

Of course, much of the sense of excitement may just have been halo effect: the girls were glad to see a different face, and be listened to. But offering philosophical debate does more than merely improve self esteem - it leaves room for each student to feel creative, to think not only algorithmically, but also about the limits of the conceptual space.

TESTING AND EVALUATION

Introducing philosophy of mathematics into the curriculum is far from easy. There is a deep suspicion of philosophers not only among mathematicians, but also, at least in Australia, among educationalists. But it also true to say that enlightened mathematics teachers have always used philosophical questions as prompts for their better students, to encourage and to baffle them. I am suggesting here that we need to use philosophy even for weaker mathematics students.

The major difficulty is of course that of testing of the students and evaluation of the program itself. Testing is an art form in mathematics: generations have devised ever-tricky problems to weed out the best students. But all testing is individual based and requires primarily a command of algorithms. The philosophical model proposed here is simply not suitable for individual testing. It is based, as I emphasised earlier, on questioning, which is dialogical. It involves capacities of reasoning, of questioning others' responses and seeking out the assumptions in what is said. It requires attention not only to one's own ideas, but also to the counter arguments others put forward.

It is precisely for this reason that philosophy of mathematics fosters P creativity. But P creativity in this sense is not readily testable. The model I advocate (forthcoming) looks to the analysis of discourse to identify suitably creative thinking patterns. Indeed, Daniel is now developing and operationalising the notions of critical and creative thinking skills so as to apply them to the transcripts derived from classes in philosophy of mathematics. But that method is to be used for the evaluation of the program, not for the testing of students. We have a long way to go before the maths test is a discussion of infinity.

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Philosophical dialogue among pupils:

A significant means to learn Mathematics

Marie-France Daniel

Previous observations in the classroom had led the researchers to realize that many pupils have difficulties in succeeding in mathematics. Why? On one hand, as Matthew Lipman advocates, the school curricula are not sufficiently «meaningful» for children (Lipman et al., 1980; Lipman, 1988). On the other hand, some studies in the field of mathematics suggest that the school system does not invite children to express ideas about mathematics nor does it favor creativity. It does not allow dialogue among peers about mathematical concepts and problems, nor the construction of mathematical knowledge by the pupils themselves (Lafortune, 1992a, b).

Based on a different way of thinking about mathematics, as well as a new way of doing philosophy, we seek to invite primary school pupils to participate in philosophico-mathematical «communities of inquiry» that will help them tame mathematics, understand better and have more pleasure in doing it. In this communication, I will present the Philosophy for children approach adapted to mathematics (P4CM).

THE P4CM CURRICULUM AND METHODOLOGY

Traditional teaching and learning in mathematics often lack the cooperative attitudes, freedom of thought and originality that ought to be developed in this subject-matter. As a consequence, pupils sometimes view mathematics as a demanding and highly competitive discipline, where only one right answer is correct and one way of solving problems is accepted. If learning as development of the whole self means anything in learning mathematics, we argue that one should privilege the involvment of pupils into philosophico-mathematical «communities of inquiry.»

THE P4CM CURRICULUM: TWO NOVELS AND A TEACHER'S MANUAL

In this perspective, the curriculum we designed includes a novel and a teacher's manual. Why a novel? We believe, relying on Lipman, Whitehead, Ricoeur, Egan and Bruner's arguments, that at school, one should begin with story. A story can make the subject come alive. Although this point of view is shared by a majority of persons, the conceptors of mathematics books seem to consider that there is too much of a gap between the objectives of literature books (fancy reading that causes feelings and emotions) and mathematics books (which provide hard information in expository style) to link one with the other. We argue that most regular mathematics books reflect too much of adult conceptions of mathematics and not enough of pupils' experiences. In this sense, the novel we wrote depicts pupils' daily life experiences in relation to philosophico-mathematical concepts and problems. The narrative is based on young people's personal concerns and expanded in such a way as to encompass mathematical and philosophical concepts and problems. These concerns are presented as exchanges between pupils akin to Socrates dialectic. As such, the novel is an attempt to generate «an organic connection between education and personal experience» (Dewey, 1938, p. 25).

The teacher's manual is essentially composed of discussion plans and mathematical activities. It aims at helping the teachers in their maieutic task. The discussion plans and activities are puzzling, paradoxical and ambiguous. We believe that pupils will enjoy mathematics if they are confronted with intellectual challenges. For children, a challenge is a game; it is what makes school an adventure rather than a routine (Lipman, 1988).

The philosophico-mathematical material concerns primary school pupils, aged from 9 to 13 years old. It should be used one hour a week within mathematics class. Its main objectives are: 1) to foster philosophical discussions among peers with regard to mathematical concepts, problems and prejudices; 2) to help pupils develop cooperative attitudes within the mathematics class; 3) to give pupils the opportunity to construct and experiment their own mathematical theories, principles and problems, in order to feel the same fascination, excitement and pride the first mathematicians felt when they elaborated their mathematical laws.

THE METHODOLOGY

The P4CM methodology evolves, like the P4C, in three steps: 1) the reading of a chapter of a novel; 2) the gathering of pupils'questions; 3) the mathematical activities and philosophical discussion among peers.

THE FIRST STEP

The first step consists in reading a chapter of the novel. The specificity of the reading is that it is done taking turns and aloud. It is a shared reading. Traditional mathematics programs do not usually involve pupils in the fostering of reading skills. Yet, when reading aloud in mathematics classes, pupils learn to establish useful relationships between various skills and different subject-matters. They get to what John Dewey calls the «principle of continuity» (1897/1972).

Also, reading in mathematics class by taking turns helps pupils develop social and moral attitudes: a) by speaking and listening, they learn reciprocity, tolerance and respect for each other; b) by sharing sentences of the novel, they experience part-whole relationships; c) by actualizing a common goal, which is to understand the story together, they learn to focus on community rather than on oneself.

Finally, while assimilating the content of the novel, pupils discover that mathematical concepts and problems can be contextualized. The context of the novel reflects not only school situations, but daily life experiences with respect to mathematics. In this sense, reading the novel helps pupils make the translation from mathematics learning at school to the resolution of personal, social and moral problems.

THE SECOND STEP

The second step of the P4CM methodology is the gathering of questions that may arise in pupils' minds after the reading. This second step is fundamentally different from traditional mathematics pedagogy. Indeed, in the traditional mathematics class, it is the teachers' privilege to ask questions. Moreover, teachers' questions are usually related to a kind of testing; they seldom lead to inquiry among pupils. We believe, along with more and more researchers (namely Davidson, 1980; Burns, 1990), that traditional mathematics teaching favours the competitive model of one question/one good answer or one question/one good way to resolve problems. Instead, the P4CM approach fosters the search for mathematical meanings within a community of inquiry. And the community of inquiry's starting point is the pupils' personal questions.

To formulate philosophical questions in mathematics is not an easy task. In fact, it is usually more demanding than to give answers, for answers may often come from memorization. Actually, the elaboration of questions engages pupils in a process of comprehension, assimilation and maturation. The conception and formulation of questions presuppose (and develop) higher-order thinking skills. Indeed, to formulate a philosophico-mathematical question presupposes that one is able to apply her or his comprehension of the mathematical novel content to more general contexts of philosophical inquiry; it presupposes that one can bring forth ambiguities, relationships, doubts, problems and uncertainties related to the field of philosophy of mathematics; it presupposes that one develops logical reasoning as well as moral judgment[2].

Let us add that this step encourages pupils to become intrinsically motivated to learn. Indeed, the second step considers that pupils are responsible for a part of their mathematics education - teachers do not plan all the programs for them; pupils are responsible to draw their own agenda of discussion[3]. In traditional settings, the teachers are the only ones who know the objectives of the mathematics program; they let pupils partially discover them as the process progresses; it is only the teachers who prepare the contexts, the experimentations and the problems to solve; the teachers's role is to supervise pupils' work and to give explanations about what they believe is important for pupils to learn (Bouchard et al., 1993). In P4CM, pupils motivate themselves to investigate and to learn; pupils are made responsible for what they will discuss about.

THE THIRD STEP

The third and last step of P4CM is based on an antique pedagogy and yet, it is revolutionary in the world of education[4]. In regard to mathematics, the last step is characterized by the philosophico-mathematical dialogue within a «community of inquiry».

A philosophical dialogue - whether it concerns mathematics, ethics, logic or esthetics - is a guided discussion which aims at helping pupils in the development of personhood [5]. Indeed, «to dialogue» is not synonymous with «to converse» or «to talk» (Reed, 1983, 1992). The term dialogue has its source in the Socratic dia-logos, which is strongly related to the pragmatic conception of «authentic communication» (Rorty, 1988/1990). Dialogue is thus understood in its sense of participation in the reconstruction of personal and social discourse. In mathematics, to dialogue involves a search for meaningful exchanges about mathematical concepts, notions, problems or attitudes, instead of preformulated answers or rhethorical argumentations. As such, the philosophico-mathematical dialogue is pluralist in its essence, for it invites pupils to share a plurality of means to solve a problem, a plurality of meanings for ambiguous concepts and a plurality of reactions towards myths and prejudices. Pluralism, here, should not be confused with relativism, for the purpose of the philosophico-mathematical dialogue is to verify the validity or the pertinence of existing knowledge, traditions, norms and values in regard with mathematics and, from a personal point of view, to verify one's own initial hypotheses, opinions and beliefs and, if needed, to modify them[6].

Within the third step, pupils practice thinking in common, they experience communicating in an authentic fashion, and learn to deal positively with interdependency.

CONCLUSION

In summary, the P4CM principles are characterized by the following: the apprenticeship of mathematics is centered on pupils' interests; it gives pupils the opportunity to actively participate to the construction and elaboration of their apprenticeship; it gives them the opportunity to become responsible (to make choices and decisions); it favours interrelationships between pupils; it develops self-esteem and fosters altruism. This does not mean that P4CM is a magical method which instantaneously transforms pupils into good persons and performant pupils. What philosophico-mathematical communities of inquiry offer to pupils are: 1) a social (non-traditional) context to learn and to do mathematics and 2) the possibility to regularly exercise (one hour a week) in living cooperative experiences on mathematics learning.

NOTES:

l. Some parts of this paper were previously published in: Daniel et al., 1995.

2. For a study of the criteria of moral judgment, one can read Lipman, 1991, 1992 and Schleifer, 1992.

3. Concerning the capability of pupils to handle their own school projects and education, one can read Nicholls and Hazzard, 1993.

4. For a more complete analysis, one can read Daniel, 1992, chapter 2.

5. One can read Sharp, 1985, 1992.

6. About the meaning of relativism from the Pragmatic perspective, one can read Bayles, 1966; Rorty, 1988/1990.

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CONCLUSION

As part of the panel, we read with the participants an extract of the novel: *The Mathematical Adventures of Matilda and David*, developed by Dr Daniel's research team at CIRADE. This novel and its manual are designed to explore philosophically mathematical concepts with children 10 or 11 years old. The extract of the novel was read in Spanish. It involves a discussion between Mark and Rosalie in which they are discussing the question of whether the number of possible moves in chess is infinite, or merely very large. They agree that there is a very large number of possible moves in chess, but a finite and definite number. Talking of the number of grains of sand, Dominic draws a distinction between an infinite number and an indefinite number, where the latter depends on our capacity to count. Finally they fix on two examples of infinity, a straight line and an increasing series.

After reading the extract of the novel with the participants, we engaged in a dialogue with them, based on their own questions, as it is done in a philosophical community of inquiry. The participants were mainly philosophy students and staff from different universities.

The question chosen was: Can infinity exist? We then proceeded to do an exercise of the manual, a discussion plan including such questions as:

Is it possible to imagine something infinitely big? Small?

Is everything unmeasurable infinite?

The group found the distinction between indefinite and infinite very difficult, and was inclined to debate its application in the case of the grains of sand. After all, the difficulty with grains of sand is that we have no clear criterion of one grain of sand. Each grain can divide. The debate then moved to the issue of how imagination is linked to infinity. Many found convincing a view which could well be labelled constructivist - that infinity is not something we imagined as complete, but was a result of the fact that there was no stopping point, that infinity just kept going on. We then turned to measuring infinity and, having detoured through

Cantor's proof that the number of integers was equivalent to the number of even integers, ended on a note of indecision. A vivid debate on the infinity of space and time continued well into the break.

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