Evolution of Pupils' Attitudes to Mathematics When Using a Philosophical Approach

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This paper forms part of an ongoing global study examining the effects of a philosophical approach in mathematics. In other papers, we have presented results concerning thinking skills (Daniel, Lafortune, Pallascio, and Schleifer, submitted for publication), and concepts of co-operation (Schleifer, Daniel, Lafortune, Pallascio, accepted for publication). This paper focuses on the impact of a philosophical approach on attitudes toward mathematics. We have previously noted (Daniel, Lafortune, Pallascio, Sykes, 1994) the existence of a perception that attitudes become more positive to mathematics when a philosophical approach is used. In this paper we present a more systematic study of a group of students from two different schools, whose initial attitudes toward mathematics varied from positive to negative. We first present our general approach, and then discuss the results obtained.

PHILOSOPHY FOR CHILDREN ADAP'T'ED TO MATHEMATICS (P4CM)

The *Philosophy for Children* curriculum applies to various fields of philosophy (logic, ethics, aesthetics, politics, etc.) (Lipman, 1988, 1991; Lipman, Sharp and Oscayan, 1980). Philosophy is considered in itself and not through other academic disciplines. Moreover, we agree with Hitchock (1992, quoted by Smith, 1995) who stresses that pupils must learn to become aware of the human element in mathematics; he suggests that pupils be regularly involved in interactive dialogue. From this perspective, teaching mathematics in elementary or high school should not consist of descriptions, explanations or beliefs, but rather of getting the pupils to discover, to invent and to validate (Daniel, Lafortune, Pallascio, Sykes, 1994, 1995; Lafortune, Daniel, Pallascio, Sykes, 1996; Pallascio, 1992). Because of the relationship that can exist between attitudes to mathematics and academic results (Ma and Kishor, 1997), we conceived (through the *Etalez* votre science program, MICST, 1993-1995) two philosophico-mathematical novels for pupils (Daniel, Lafortune, Pallascio, Sykes, 1996a-b) and an accompanying guide for teachers (Daniel, Lafortune, Pallascio, Sykes, 1996c). Both novels are adapted to the ages of pupils in the second half of elementary school or in junior high school. They relate to philosophical concepts and mathematical notions (infinite vs. indefinite, zero vs. nothing, etc.). The novels also relate to beliefs, attitudes, stereotypes and biases often conveyed by youngsters in this age group (Lafortune, 1993). The accompanying guide is intended to assist teachers in their Socratic role or *maieutic.* The guide provides teachers with suggestions for discussion plans or mathematical activities related to the novels' main themes.

In this approach, mathematics takes on a human and social dimension. It is presented to pupils as a construction of the human spirit to which each generation contributes, rather than more-or-less magical data «discovered» by «scholars». This approach proposes a vision of mathematics that enables pupils to become aware of the social aspects in the production of mathematical knowledge.

In the *Philosophy for Children* approach adapted to mathematics, we make full use of the three steps defined by Lipman and Sharp. We add in moments of individual reflection, as well as particularly mathematical or scientific activities (group or individual), according to the situation. Clark (1994), who adapted the *Philosophy for Children* approach to biology education, inspired our use of moments of reflection. Individual reflection prior to discussion allows the pupils to take some time to think about and organize their points of view. We believe that more pupils are motivated to engage in the discussion when they reflect individually before approaching the community of inquiry. Indeed, they can each develop their own opinion, take a stand and propose relevant arguments to defend their points of view. Individual reflection following a philosophical discussion allows pupils to make their own assessments. It is not so much a matter of providing an answer to a specific question, but rather of summarizing and prioritizing the elements that emerged from the discussion in the community of inquiry. Whether this step be written or oral, we believe it is a logical consequence of any well-led community of inquiry. Indeed, the community of inquiry's aim is to arouse significant questions in the youngsters' minds, to awaken their curiosity and to motivate them to pursue the research process begun in the classroom. It is a matter of deciding on the relevance of this process and the manner in which it will take place according to the contents of and progress in philosophical discussions experienced by the pupils.

Some of the exercises or activities to be performed by the pupils focus on mathematical notions that form an integral part of the mathematics class (e.g.: the difference between number and digit); others focus on the myths surrounding mathematics and sciences (e.g.: *superior talent is necessary in order to succeed);* still others focus on the affective aspects influencing learning in these disciplines (e.g.: fear of failure in mathematics). Some of these activities rely mainly on philosophical discussion as an approach to mathematics and science, whereas others focus on the disciplines themselves in order to provoke philosophical discussions.

The *Philosophy for Children* approach adapted to mathematics is new and promising. The first observations from teachers and the research team, stemming from validation of the novels and guide process, lead one to suppose that pupils can develop more positive attitudes toward mathematics while being more critical about their beliefs and biases. There also seems to be a better communication of pupils' philosophico-math-ematical ideas and an evolution of pupils' reac-tions toward mathematics, and competition seems to give way to a search for equality among group members. We were therefore forced to consider the importance of the roles of philosophical training and co-operative practice for meaningful learning of mathematics, and to investigate the contribution of this approach to the attitudes of some pupils toward mathematics. We explore the latter aspect in this paper.

ATTITUDES TOWARD *MATHEMATICS*

Mathematics teaching does not always allow pupils to develop positive attitudes toward the discipline, and it seems that the more pupils progress in their studies, the stronger the relationship is between their attitudes toward mathematics and their academic results; in other words, at the beginning of junior high school, for example, if attitudes toward mathematics are positive, good academic results are likely in future. In a meta-analysis, Ma and Kishor (1997) stress that between the first four years and the last two years of elementary school, the relation between attitudes toward mathematics and academic performance increases by 367%, that it increases by 79% between the end of elementary school and the beginning of high school, and that it diminishes by 20% between the beginning and the end of high school. In our view, it is therefore important to intervene as early as elementary school, so that pupils do not develop too many negative attitudes toward mathematics, and can thus enjoy increased success in this discipline.

Numerous studies undertaken on attitudes toward mathematics (for example, Armstrong and Price, 1982; Reynolds and Walberg, 1992), show that pupils' attitudes often become more negative during the passage from elementary school to high school. It is therefore important to consider the transition between these two types of teaching. To these results we add the research of Nimier (1976, 1985), of Blouin (1985, 1987), of Lafortune (1987, 1988, 1990, 1992a, b), of Tobias (1978, 1987, 1990) and of Baruk (1973, 1985), showing the importance of intervening in the affective dimension of mathematics. For example, a pupil who begins class convinced that she/he does not have the capacity to succeed in mathematics cannot open her/his mind to the explanations being given. Tobias' (1978, 1987) and Lafortune's (1987, 1988, 1990, 1992) research focused on the particular situation of adults whose memory of traumatizing events in elementary or high school made it a real nightmare for them to pass a mandatory mathematics class when going back to school. The situation of these adults shows just how important it is to intervene as early as elementary and high school studies. Along the same lines, Meece, Wigfield and Eccles (1990) specify that numerous studies (Armstrong, 1985; Betz 1978; Brush, 1980; Eccles [Parson], 1984; Hackett, 1985; Hendel, 1980; Richardson and Woolfolk, 1980; Sherman and Fennema, 1977; Wigfield and Meece, 1988) quoted by Meece et al. (1990), showed that affective reactions toward mathematics negatively influenced youngsters in their choice of advanced mathematics after their high school studies.

Even if teachers are concerned with linking the cognitive, affective and meta-cognitive dimensions of learning, they have few means at their disposal to integrate these dimensions (Lafortune and St-Pierre, 1994, 1996). It therefore becomes difficult to attain objectives such as the following: to approach a mathematics problem with an open mind, to react positively to one's errors or to benefit from them, to persist in the search for a solution. To disregard pupils' affective reactions toward mathematics is to lose an opportunity not only to avoid failures and withdrawals, but also to share a love for mathematics and to transmit a more realistic perception of mathematical work.

To explore ideas conveyed on mathematics by youths aged 7 to 13, Lafortune (1993) undertook a research project in which youngsters (17 youths, 8 girls and 9 boys, separated into three groups) were asked to «draw mathematics». The drawings they produced were quite surprising. One pupil chose to represent mathematics with the teacher in the shape of a horned devil with a forked tail, another drew a young boy crying in front of his math copy, one pupil feels incompetent and describes mathematics as a «sequence of mistakes», yet another drew a pupil thinking about mathematics by adding a balloon with a youngster sitting on a mountain surrounded by flames (mathematics seem to represent hell), etc. To help both the researcher and their classmates to understand the drawings, the youngsters agreed to participate in a group interview (3 interviews per age group; group sizes 7 pupils, 6 pupils and 4 pupils) to present their drawings to the others, to explain the choice of contents and to answer other pupils' questions.

In the analysis of the content, two types of myths and preconceived ideas conveyed by these elementary and high school pupils emerged. Firstly, there were myths and biases that concerned mathematics in itself. We refer to widespread ideas such as *math is useless, math is nothing but arithmetic* or *geometry isn't math.* Then, there were myths and biases that refer mostly to the learning of mathematics. Along these lines, we encountered such expressions as *brainies are boring, nerds and stuck-ups, those who don't understand don't have any logic* or you *can't study math.*

This situation leads certain pupils to develop negative attitudes toward mathematics. Some enter the classroom convinced they will be bored and won't enjoy the subject at all. For others, math class is a burden to be endured; they come to ignore what is said during the class, and thus encounter difficulties in achieving a passing grade. Others are convinced from the start that they will fail, as they attribute success in mathematics to the possession of some special talent, or believe there is such a thing as *baving a good head for math.* Consequently, these pupils have an excellent pretext to explain their failures or to convince themselves that any effort on their part would be futile. Finally, other youngsters foster «misbeliefs» and perpetuate them among their schoolmates. For example, they believe that *math is magical,* and they therefore cannot see problem-solving as a search for a solution involving time, reflection, error and effort. The end result of all these myths and biases is the difficulty of many pupils in accepting responsibility for their learning, or in believing they have control over their academic results.

More and more, research on teaching mathematics explores approaches that require pupils to be more active in their own learning process. Instead of being knowledge-receptors, they can question and reflect on mathematics, or on their comprehension of mathematics. The results of Brush's (1997) and Leikin and Zaslavsky's (1997) research experimenting with co-operative learning, that of Choi and Hannafin (1997) who used contexts in problem-solving, and of Di Pillo, Sovchik and Moss (1997) who experimented with a learning journal, show that pupils were able to share their ideas regarding mathematics in order to better integrate their learning and find a means to communicate with their teacher. Many studies (Meravech and Kramarski, 1997; Petit and Zawojwoski, 1997; Jitendra and Xin, 1997; Anthony, 1996; Goos and Galbraith, 1996) have shown that interventions using non-traditional approaches (meta-cognitive activities, peer interaction, problem-solving, appropriate use of technologies) allowed pupils to develop positive attitudes toward mathematics, or to achieve better academic results.

In this research project called *Philosophical Education and Co-operative Practices in the Learning ofMatbematics* (Daniel, Lafortune, Pallascio, Schleifer, funded by a grant from HSRCC [Human Science Research

Council of Canada], 1995-1998), we pursue the following objectives: to study the extent to which the model of co-operative teaching we favour in mathematics influences pupils' attitudes with regard to the subject matter; to study on a cognitive level whether pupils develop higher-order thinking skills about mathematics following philosophical practice; and to study the evolution of the concept of co-operation in pupils following weekly communities of philosophical inquiry. In this paper we present the results concerning attitudes toward mathematics.

METHODOLOGY

For this research project, we collected both qualitative and quantitative data. In this paper, we present the results of qualitative data collected from two class-groups on which we conducted interviews and made typical case observations.

The qualitative instruments used to collect data from four typical pupil-cases can be summarized as follows: the drawing representing mathematics (pre-experiment and post-experiment) and a brief explanation of the drawing (1 to 3 sentences); observation grids completed by the teachers (during the regular mathematics classes) and by the research assistants (during the communities of inquiry on mathematics), and individual meetings (post-experiment).

Teacher training, along with validation of the instruments used for data collection, took place in 1995-1996. Systematic collection of data took place in 1996-1997. Data analysis was carried out in 1997-1998.

The Drawing

To create their drawings, pupils were asked to «draw mathematics», and to write on the backs of their drawings 1 to 3 explanatory sentences. The drawings were done individually. Pupils were informed that the quality of the drawings was not a priority. To give their imagination as much leeway as possible, we insisted that all their ideas were valid, whether they seemed closely related to mathematics or not. We used the first drawings together with their explanatory sentence(s) to choose the typical pupil-cases who would be subject to observation for part of the school year, and who would be interviewed at the end of the experiment. For this choice, the drawings were divided into four categories: 1) very positive drawings; 2) somewhat positive drawings with some negative aspects; 3) somewhat negative drawings with some positive aspects; 4) negative drawings.

Two researchers sorted the drawings for the same group of pupils. Each researcher sorted the drawings individually, using both the drawings and the explanatory sentences to choose the category to which the drawing corresponded. The researchers initially agreed on more than 80% of their compilations. Following this, they compared compilations and discussed their choices to reach a final consensus.

Compilation of the drawings undertaken at the beginning of the year yielded the following results for the two school-groups:

Location	Very positive drawings	Somewhat positive drawings with negative aspects	Somewhat negative drawings with positive aspects	Very negative drawings
1	1 boy 4 girls	7 boys 11 girls		
2	4 boys 1 girl	1 boy 2 girls	2 boys 3 girls	4 boys 4 girls
TOTAL	5 boys 5 girls	8 boys 13 girls	2 boys 3 girls	4 boys 4 girls

From this compilation, we identified eight pupils (four per school-group). We chose two boys and two girls per group, plus two pupils who showed a strong appreciation of mathematics and two others who showed less appreciation.

Observation Grid

All eight typical pupil cases were observed during the research, mostly during the second half of the year. An observation grid for each pupil was completed weekly by both the activity leader accompanying the teacher after the *Philosophy for Children* adapted to mathematics meeting, and by the teacher from the mathematics classes that took place during the week. The observation grid consisted of the following observations: attitudes with regard to mathematics: positive aspects (manifestations of pleasure, etc.), negative aspects (manifestations of rejection, etc.); co-operative attitudes with regard to the tasks involved (respect, hearing others out, participation, mutual assistance).

Individual Interviews

Using one of the two drawings (pre- and post-experiment which for commodity we will call pre- and post- drawings) produced by the pupil (generally, the second drawing, which the pupil remembers best), we prepared the protocol for individual meetings. This protocol consisted of the following manner of questioning. The first questions were intended to determine why the pupil had chosen to represent mathematics in whatever way she/he had done, and whether the drawing in question represented mathematics in a positive or negative manner. Other questions were designed to figure out$_s$ the link between the drawing and its explanatory sentences. These also attempted to determine whether or not the pupil would change the drawing if she/he could start all over again. The final questions were asked to find out whether or not the pupil had profited from the philosophical communities of inquiry on mathematics.

In this paper, we present the evolution of attitudes toward mathematics in pupils originating from two different locations: a sixth grade classroom (Location 1) and a fourth grade classroom (Location 2), both in the Montreal area.

RESULTS ON THE EVOLUTION OF PUPILS' ATTITUDES TOWARD MATHEMATICS

Here we present a synthesis of the negative results from the data compiled for the 8 typical pupils who were closely studied in the school environment. First, we present a synthesis of the data from the 4 case studies at the first location (El, E2, E3, E4), as well as a synthesis on this location as a whole. We proceed in the same manner for the second location (E5, E6, E7, and E8).

Case Study at Location 1: Sixth-grade Class

The pre- and post- drawings of E1 (boy) do not necessarily show the evolution of what this pupil thinks of mathematics. Prior to the experiment, this pupil already had a positive view of mathematics. In his first drawing, he traced personalized mathematical symbols; he drew division and multiplication signs for the mouth. Other mathematical symbols have eyes, a nose, hair, arms and legs. A frame in the backdrop represents addition and multiplication symbols in the form of a photograph. Mathematics is represented as though it were a form of life. He explains his drawing with the following comment, *«It represents what math and symbols mean to me. «* In his second drawing, not only did he represent mathematics with symbols; he also added equalities using these symbols. This second drawing presents a greater diversity of elements from the idea the pupil has of mathematics; the symbols are related to numbers or letters: 4(6+24, E14, etc. The following sentence explains his drawing, *«This drawing means equations. It's what makes me think of math. «*

It is evident that E1 already had positive reactions toward mathematics; the researchers who observed him were unequivocal on the subject. However, it seems that El did not perceive on his own that he had good capacities in mathematics. P4CM seems to have allowed him to become aware of his abilities both in mathematics and in philosophy, *«I have now become good, I don't need my family.»* P4CM seems to have allowed him to realize his comprehension of mathematics: «[. ..J *when I went to math class, I found it more difficult while as [now], it is [easily] done. [.J 1 am among the three best in my class; last year 1 was among the worst. «* P4CM has also allowed him to reflect on mathematics: *«Everybody should get a chance to do philosophy; [...] everyone deserves to have philosophy; it's super «cool» to help you understand and reflect.* «According to E1, this new awareness will allow him to reduce his fears toward his studies in high school; he will have strengthened the clarity of his expression and of his coherence; he will have learned to follow through on his ideas. He therefore feels he will be better able to defend his ideas when in high school.

The pre- and post- drawings of E2 (boy) show a certain evolution in what this pupil thinks of mathematics. Even though this pupil had a positive concept of mathematics prior to the experiment, he seems to have expanded this positive concept. In his first drawing, E2 represented mathematics in the guise of a character composed of geometric figures: circles for eyes and mouth, a triangle for the nose and two rectangles for the cheeks. He adds a fractionated rectangle and equalities containing numerous equations. He uses many colours to illustrate the different elements of his drawing, and adds a question mark and an exclamation mark. The following sentence explains his drawing, *«Math reminds me of all the things I illustrated.* «In his second drawing, E2 portrays two soccer players. In the upper left-hand corner, he drew a big sun surrounded by flower-clouds. He added the following sentence to explain his drawing, *«To me math is like a game, a game I'm really good at and [in which] I like to distinguish myself from others. «* Even though E1's first drawing is quite positive, it depicts mathematics in a traditional manner. The second drawing shows a change in this pupil's vision of mathematics: it is perceived as a game. The traditional aspect has disappeared.

It seems that P4CM helped E2 gain some ease with mathematics and a better appreciation for the discipline. He discovered he enjoyed doing mathematics and discussing the subject; this could explain his second drawing, portraying soccer players, compared to the first drawing representing traditional aspects of mathematics (formulas, equations, numbers, etc.).

In the interview, E2 mentions that P4CM taught him to ask questions and thus acquire a greater ease in mathematics. It helped him develop his confidence to succeed in mathematics. Before, he was sometimes afraid of making mistakes. In P4CM, error is just about non-existent, since all pupils are aware that their reflections are open to challenge or question so they can be improved, corrected and modified, without penalty. Also, by asking more questions, he probably learned to search for solutions. In addition to discovering more pleasure in doing mathematics, E2 learned the meaning of philosophy, «[as *the year progressed...] I understood better, [therefore] I liked going to philosophy much more; [it] really helped me to think in math, [..] math has become a game [which] I like to play»*. It seems that E2 made various discoveries by engaging in P4CM: the game aspect of mathematics; self-confidence regarding his possible mistakes, («I *didn't ask enough questions, [...] was almost never sure of my answers, but with philosophy, I trust myself I've become super good at math.');* the questioning and the search for solutions, *(«[that/ helped me to ask more questions, to always search for solutions, to try to transform math [into] problems, situations that [can] happen to us on a daily basis.').*

The pre- and post- drawings of E3 (girl) show a certain evolution in what this pupil thinks of mathematics. Even though prior to the experiment this pupil did not have a totally negative concept of mathematics, her representation of mathematics seems to have become more positive. In her first drawing, E3 drew a calculator with numbers and equation signs of different colours. She explains her drawing by writing, *«My drawing represents the numbers and symbols of a calculator. «* Her second drawing shows more diversity. Balloon shaped clouds containing words such as: problems, solids, multiples, solutions, shapes, questions, vertex, geometry, divisor, face, denominator, numerator. What is most significant is the sentence explaining this drawing, *«It is a person thinking about math, by saying all that comes to her mind. words and symbols»*.

E3 does not say so explicitly, but we can infer from her remarks that P4CM played some role in improving her comprehension of mathematics, « [...] *now, I am more used to doing problems; I'm not that good, but I'm able to do some. I'm able to solve at least two out of three. Before, I couldn't solve [any].* « The approach seems to have contributed to teaching her to solve problems.

Even though she hardly participated in the exchanges within the philosophico-mathematical communities of inquiry, according to her teacher, she actively listened to the discussions. P4CM did not bring her to express herself more freely in a large group, but perhaps it allowed her to realize that her way of studying wasn't adequate, *«I sure changed my way of studying; I study math about an hour every day [..], before, I would do fifteen or thirty minutes; I didn't really like it (mathematics). «* E3 seems more at ease in expressing herself in small groups, according to her teacher.

The pre- and post- drawings of E4 (girl) show a certain change in this pupil's way of thinking about mathematics. Although prior to experiment she did not have an extremely negative concept of mathematics, her representation of mathematics now seems more positive. The first drawing is dynamic and filled with colour. In it we find fractions, sums, numbers and symbols of mathematical equations. There are also geometric shapes, a calculator and a math book titled «Daily Math». The second drawing is just as dynamic. It contains many elements. There are five characters. For example, a pupil puts a poster of a sum up on the wall; another holds a platform on which two others hold onto each other by the shoulders and hold up a poster picturing a fraction. Frames show words such as: multiplication, number with comma, equation signs, etc. On the drawing we also find other elements such as a calculator, two cubes, etc. This pupil explains her drawing with the following sentence, *«My drawing represents a few people trying to do or rather representing math. They are putting up a few posters relating to math: algebra, +, =, (, +. «*

Although the first drawing is dynamic, it has only symbols to represent math; no people seem to be linked to mathematics. In the second drawing, mathematics become less abstract. Real people are busy doing math. E4 changed her representation of mathematics a little by associating it more with people, rather than representing it in a completely abstract manner. As she says, by doing P4CM, «I learned to think [about mathematics], I learned to speak what I think and what *I like.»* Even if this pupil wasn't always paying attention during the communities of inquiry, on various occasions she showed logical reasoning. Her teacher even says she ensured the proper functioning of various communities of inquiry.

GENERAL SYNTHESIS OF LOCATION 1

In this location, where the teacher had a passion for mathematics and for P4CM, the pupils had a positive experience of the P4CM approach. Many of the pupils interviewed explicitly mentioned the contribution of P4CM to their evolution regarding mathematical problem-solving. This contribution was manifest on the affective level (boost in self-esteem, lessening of fears regarding mathematics and error), on the cognitive level (ability to reflect, to question, to solve problems, attitudes of perseverance, rigour) as well as on the social level (capacity to communicate one's ideas to peers, respect for differences of opinion).

Case Study of Location 2:

Fourtb-grade Class

The first and the second drawing of E5 (boy) show equally that he likes mathematics. In his first drawing, E5 drew a little man with a big red smile that takes up the foreground of the drawing. He wears a multicoloured suit. This little man states a multiplication problem in a balloon. The character seems to be in full action, arms in movement, legs apart. To this character are added mathematical signs such as addition, subtraction and multiplication. In the background, another character states a mathematical problem containing a sum, and an equation is written on a long green suspended chalk-board. E5 describes his drawing with the following sentence, *«I love math, I'd like to do math all day long»*. In his second drawing, E5 didn't draw any characters, but the presence of people is felt, for he drew balloons containing various sentences. He drew a long chalk-board with a division and a fraction with the corresponding decimal fraction; he added the words Cool *Math*. A balloon suggesting the presence of the teacher says, *«Open your exercise books to page 61 «*. Two other balloons suggest the presence of pupils who answer, *«Math is cool»* and *«True»*. An addition, a division and a subtraction sign complete the drawing. The following sentence explains the drawing, *«I really like math. I like to learn, that's what this drawing means. COOL.))*

E5 already liked mathematics prior to engaging in P4CM. He retained his passion for mathematics, and prefers mathematics to P4CM. His love of mathematics seems to have had an influence on his involvement in P4CM, as he appears to have participated actively in the communities of philosophical inquiry.

E5 is a boy who likes to learn; he says so himself, *«[sometimes/ when we went to the grocery store, I'd write things down, that's math. I'm not aware of it, but I do it a lot... «* His preference for mathematics comes from the impression he has of learning more in mathematics: *«the book is bigger, that means there are more things to learn. «* However, the comments made by those observing him seem to indicate that he really liked the communities of inquiry in mathematics. He showed a co-operative attitude by actively participating, by wanting the others to understand and by sharing his chain of reasoning.

E6's (boy) two drawings are very similar. This pupil's pre- drawing shows a round head, without a body; fire coming out of both ears, two mathematical signs written in balloons on each side of the ears; big mathematical signs surrounding the bottom of the face. A black cloud glides above the head and lightning comes out of each end (left and right) of the cloud. During the interview, the pupil stressed that this drawing was a self-portrait.

E6 explains his drawing with the following text:

1. My drawing shows my dislike of mathematics.

2. My drawing shows that I'm all mixed up.

3. My drawing shows that I'm angry...

In large letters at the bottom of the page, he wrote, *«FORBIDDEN TO MATH»*. The post- drawing also shows a round head, without a body, all red, with bulging eyes and fire coming out of the ears. The mouth is open and in a balloon are the words, *«1 hate math»*. There is a gray and blue cloud above the head; there are raindrops between the cloud and the head, and on each side of the cloud, lightning and rain. Another balloon in the right hand corner above the head illustrates a thought: it is a self-portrait (the pupil confirmed this during the interview) that shows him jumping on a book on which mathematical signs have been drawn. The text that explains the drawing reads as follows, *« My drawing shows that I hate math because each time I see math, I have to stop myself from getting angry. «*

It seems E6 (boy) doesn't like mathematics, and even hates it. However, it doesn't seem as though the activity leader or the teacher 'realizes this. According to his teacher, he hardly participated in the philosophical communities of inquiry. Also, his teacher's interpretation of his promptness at completing the required math exercises doesn't coincide with what E6 says, *«I work fast, so that I can hurry up and do something else. «* For him, going fast is a way of getting rid of his math. He only slows down when necessary to avoid failure. The P4CM doesn't seem to have helped him appreciate mathematics. From the beginning to the end of the school year, he was only slightly or not at all co-operative during the proposed activities in the P4CM sessions. This reflects his lack of interest as much for mathematics as for P4CM. Pertaining to his logical reasoning, those observing him consider him to be capable of it, but note that he rarely manifests the ability.

The pre- drawing of E7 (girl) shows at its centre a big green chalk-board on which is written a mathematical problem including a subtraction. The teacher is standing by the blackboard and says, *«We're doing math «*. A pupil (her self-portrait according to her) is seated in front of the teacher. On the teacher's desk behind the pupil, a plant and a pencil holder are illustrated. In the middle, above the drawing, there is a devil with red horns, tee-shirt and tail and a black fork. This devil is behind a black cloud. On either side of this cloud and of the devil, there are two clouds with the following statement: *«Math is boring. «* The drawing's explanation includes the following sentence: *«I hate math. I like French a lot better. «* The post- drawing doesn't include any characters. E7 drew a book with the title *Math* written in large stylized and coloured letters. There are mathematical symbols and letters on the book. The remainder of the page is taken up with a desk on which rests a math book from which mathematical symbols, geometric forms, letters and the words *pair* and *polygon* emerge. Other letters, mathematical signs and geometric forms (in a bigger format) are drawn. To explain his drawing, E7 wrote, *«My drawing shows a math book that loses all its letters and numbers»*.

E7 seems to have evolved toward more positive reactions regarding mathematics: *«Before, I didn't really like math because it was hard; I gave up often when it was difficult. Now, I try and I like it. «* However, E7 doesn't attribute this to the *P4CM* sessions; she feels that it was due to her mother's additional help. On the other hand, the people observing her noticed an evolution in her attitude toward both mathematics and *P4CM*. Should the *P4CM* continue, it is felt that she could develop better logical reasoning and participate more in the community of inquiry.

The pre- drawing of E8 (girl) shows a pupil sitting at her desk (during the interview, she confirmed it was her self-portrait); she is smiling, holding a pencil and appears to be writing. To the right of the drawing, we notice a sum of pink hearts (one heart + two hearts = three hearts). There is also another sum. Large numbers, letters and mathematical symbols gravitate around these three main elements. Just about everywhere on the page, E8 used rubber stamps to include shapes such as hearts, footprints, spirals and stars. The sentence explaining her drawing reads as follows: *«1 really love math. I like that it's so surprising. It's funny and adorable. «*The post- drawing shows a haloed angel floating in the middle. This angel is above a pupil seated at a desk with pencil in hand. She has a black question mark on her head. There are large mathematical symbols in various colours; there is also a rectangle with a question mark inside. To explain her drawing, E8 wrote, *«In a way, I still like math. But in another, I hate it. «*

Just as E8's ideas seemed muddled during the class community of inquiry, they were equally unclear during the meetings. Ultimately, we are unsure whether E8 likes mathematics or not. If we rely on her drawings, we are led to believe she rather likes mathematics, but then, her remarks end up confounding us: *«I didn't like it (mathematics) but 1 was always able to get good grades. 1 still don't really like it. I try to listen to the teacher as much as 1 can, but I can't remember everything. «* During the communities of inquiry, she was very active, but she didn't always help the group in their progress to reach their goal. We cannot really say that there was an evolution pertaining either to mathematics or to *P4CM* for this pupil.

GENERAL SYNTHESIS OF LOCATION 2

The synthesis of the case studies from this class-group indicates that it is rather difficult to affirm that *P4CM* had a real positive influence on this group's reactions toward mathematics. One pupil retained his passion for mathematics and contributed to the philosophical community of inquiry in mathematics. Another continued to loathe mathematics, another ascribed her positive evolution in mathematics to her mother, and a final one was confused, so we cannot say whether or not she *likes* mathematics or *Philosophy for Children* adapted to mathematics.

DISCUSSION

For these two class-groups, we notice a very different evolution regarding mathematics. In the first location, the pupils really seem to have developed positive attitudes toward mathematics, whereas in the second location the attitudes, at most, remained the same. How can we explain this situation?

First, we would like to highlight the fact that this data was analyzed using a case-study process in which the data issued from pupils in groups of four. These pupils cannot represent the evolution of all groups as a whole. This data can only serve as an indication, and help us understand the evolution of attitudes toward mathematics when experimenting with *P4CM*.

The difference in the two locations could be explained by the different types of teacher leading these two groups. The teacher in the first location liked mathematics and loved *P4CM*. The other teacher didn't really like mathematics and wasn't always comfortable with the use of *P4CM*. This situation also had repercussions on the pupils' interactions during the communities of inquiry. We analysed the data issuing from the communities of inquiry in Daniel, Lafortune, Pallascio and Schleifer (submitted for publication).

The difference could also be explained by the fact that the two groups did not start off with identical attitudes toward mathematics. Compilation of the drawings shows that 100% of the pupils from the first location adopted somewhat positive attitudes toward mathematics, even though certain elements of these drawings may have been negative or neutral. The pupils from the second location didn't really *like* mathematics at the beginning of the experiment. Only 38% of the drawings showed positive representations of mathematics. One could believe that interventions of *Philosophy for Children* adapted to mathematics, occurring within a single year (mid-October to the end of April), and at the rate of only one hour per week, are not sufficient to change negative attitudes established over a number of years. Moreover, if the teacher in the second location did not really like mathematics, the interventions that took place during the rest of the week may not have had such a positive impact on the pupils' attitudes. In the first group, the teacher may have provoked positive attitudes all week long.

If many pupils from the same group didn't like mathematics (Location 2, for example), we may be justified in thinking that more than one hour per week of intervention would be necessary to influence their attitudes toward mathematics, unlike cognitive development, which seems to develop much more quickly. Indeed, both groups of pupils developed higher-order thinking skills. The following results stemming from another data analysis in this research confirm this fact (Daniel, Lafortune, Pallascio, and Schleifer, submitted). In relation to the development of higher-order thinking skills the frequency of pupil interventions rose from 9.2% to 34.6%, in the first location while those of pupils in the second location rose from 18.4% to 49%. The development of higher-order thinking skills may be the starting point for pupils' development of thinking skills related to their attitudes toward mathematics. For pupils to develop positive attitudes toward mathematics, it is likely that interventions that focus on the affective dimension during the math period will turn out to be an interesting, and perhaps even an essential, complement.

CONCLUSION

To help pupils develop positive attitudes toward mathematics, we have developed a philosophical approach to mathematics based on the *Philosophy for Children* approach. In order to verify the influence of this approach on the attitudes of pupils toward mathematics, we studied the evolution of certain pupils with the help of drawings, explanatory sentences, an observation grid and interviews.

The results of this research allow us to postulate that this approach could, in the short-term, develop positive attitudes toward mathematics if the teacher and the pupils are already favourably disposed toward mathematics (like Location 1). If such is not the case, the *Philosophy for Children* approach adapted to mathematics must be accompanied by a particular context: the teacher herself should adopt a positive attitude toward mathematics, P4CM should be used for more than one hour per week or a focus on the affective dimensions of interactions in mathematics should be integrated into the teaching of mathematics.

Since we studied the evolution of only four pupils in each class-group with regard to this aspect of the research, it would be necessary to study the evolution of a larger group in relation to various aspects of the affective dimension toward mathematics. Another research project that is already underway will allow us to achieve this objective. This research, titled *Philosophic Fxperiences in Mathematics: A Study of the Evolution of Affective Factors* (Lafortune, Daniel, Pallascio, Mongeau, Schleifer, funded by HSRCC 1997-2000) proposes to study the evolution of various elements of the affective dimension influencing the learning of mathematics: attitudes, concept of self, anxiety and causal attributions, by comparing pupils in an experimental group to those in a control group. These affective factors are situated in relation to the perceptions pupils have of mathematics. We have already developed and validated quantitative and qualitative instruments. This experiment is currently underway (1998-1999).

Following these results, a few questions remain: Should the P4CM approach be used as a complement to other reflexive approaches in order to impact the cognitive, social, affective and meta-cognitive dimensions in the learning of mathematics? Should we study the influence of P4CM on pupils for more than a year in order to have better knowledge of this influence on the various dimensions of learning? Should we favour the transfer of certain principles of the P4CM approach to the mathematics class? If so, in what way?

To partly answer this last question, for example, the *Philosophy for Children* approach adapted to mathematics could favour the use of a meta-cognitive-constructivist approach during the math period (Lafortune, 1998) which means that the pupil might be better able to ask himself questions without waiting for the teacher to ask them; he might improve the structure of his knowledge in an active manner, he might be more aware of his mental processes and of different means of self-evaluation. It might allow him to discover what it means to understand mathematics. In a meta-cognitive approach, the following means could allow pupils to transfer what they have developed in P4CM: to discuss another person's mistakes or reasoning, to compare statements and solutions and classify problems according to their level of difficulty (Lafortune, 1998).

We hope that this research will provide us with information about P4CM that will help us improve the approach for an increase of knowledge construction, abilities and positive attitudes toward mathematics.

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