

# *Critical Thinking, a Philosophical Community of Inquiry and the Science/Maths Teacher*

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*"It is far better to be engaged in making meaning of facts than to be verifying their truth."*

— Greg Smith

*"We want students to think for themselves and not merely to learn what other people have thought."*

— Matthew Lipman (1988)

*"The notion that mathematics is cold-blooded and stories are warm-blooded must be rethought."*

— Borasi (1990)

**I**n life reason and emotion are best when complementary. In fact active inquiry in philosophy and in science and mathematics do meet best in the learning process. Since significant meaning making activities can occur in mathematics, philosophy should be studied in the science class. "The notion that mathematics is cold-blooded and stories are warm-blooded must be rethought." (Borasi, 1990:188)

Philosophy is both content and process. So by doing philosophy, science students could consider authentic problems in a context that values the thinking process with an active and critical stance. For philosophy offers enriched content, processes and products. "Thinking skills must be taught as warp and woof of the disciplines" (McPeck, 1983:307). This paper argues that philosophy enhances the learning processes and it offers ways to use philosophical methods to enrich content learning in the Science and Mathematics classes.

Philosophy values thinking skills across subject areas. Reed (1985) advocates that "philosophy should illuminate and be connected with other parts of the curriculum. ...[so that children] will see the connections and so become better artists or better mathematicians" (230). "Philosophy for Children is an instrument for making the existing curriculum (whatever that curriculum may be) work more effectively." (234). Truly, they could

be thinking their way through the curriculum. So the science/maths teacher teaching philosophical-ly will make those connections and ensure that skills are more likely to transfer across to other areas of the curriculum.

Philosophy is not just critical thinking. While critical thinking may be philosophy's most teachable part and most noble achievement, it is by far not the whole of the undertaking. Imagery, ethics and aesthetics, and dialogue and communication are wider and richer domains within its proper study (MacColl, 1992). This paper is offering ways of introducing such philosophical activities in the science and maths curriculums.

### CRITICAL INQUIRY

What does philosophy offer the Science or Mathematics student? Of course, closure will be sought in Science because solutions are required, e.g., light seen as both wave and particle requires two solutions. But critical issues in society and the world are inevitably discussed in senior science classrooms already. Issues like euthanasia, IVF, the possibilities and ethics of bio-technical science, nuclear power (Gill, 1994), and so on are readily discussed. Under these circumstances, Science teachers can look to philosophy for a suitable model for running open ended discussions.

Philosophy will enrich the Science curriculum by offering a comprehensive inquiry method and practice in those very essential inductive, inferential and deductive thinking skills. Philosophy in junior science classes too will open the gates to wider content and invigorate thinking processes. Philosophy offers lower secondary students a chance for richer learnings, more relevant studies and more creative input by the learners themselves. With it, a student-driven agenda is possible.

Borasi (1992:180) highlights these crossovers when she discusses the differences between what she describes as a transmission teacher and an interpretation teacher. In transmission, a teacher's role lies in imparting public knowledge with its content and criteria of performance in direct pedagogical ways. A learner's performance is valued insofar as it conforms to the criteria of the discipline. The teacher's task is the correction of this performance and the learner must qualify herself through tests.

She advocates the interpretation model however as much more preferable and more amenable to

the philosophical method. In the interpretation model, the teacher believes knowledge exists in the knower's ability to organise thought and action; she values the learner's commitment to interpreting reality. The teacher's task is to set up dialogue and interaction to shape this knowledge, and the learner is seen as already possessing relevant knowledge and skills that can be reshaped to these new purposes.

Borasi's plea for a humanistic inquiry approach, building on these views and approaches, is a call for a revolution in mathematics teaching. Her prescription for constructive learning environments (181), with agendas where students "make decisions on their own about what they consider worth pursuing" (182), "demanding more of students in terms of initiative and responsibility, generating and evaluating hypotheses within a learning community involving more risk taking, both intellectually and emotionally" (182) is similar to the community of inquiry method of philosophy. These environments open up exciting pedagogical possibilities for including philosophy in maths and science classrooms.

### GROUP INVESTIGATIONS

Typically scientists (and inevitably science students) have a greater interest in relationships in the physical than in the psychological world. Gornick (1987) describes their outlook: "Whatever a scientist is doing — reading, cooking, talking, playing — science thoughts are always at the edge of the mind. They are the way the world is taken in; all that is seen is filtered through an ever-present scientific musing." A scientist is always testing for patterns, for intelligibility. This focus could be described as "the scientific mind" and is even parodied as "Prove it to me!"

So to cater for such different learning styles these days, a mix in classroom learning activities in mathematics is highly recommended. The famed Cockcroft Report in the UK (1982) in its most quoted paragraph (243) outlined that the desirable 'balanced diet' of learning activities involved six elements including personal investigation: "Mathematics at all levels should include opportunities for: exposition, discussion, practical work, consolidation and practice of fundamental skills and routines, problem solving including applications to everyday life, and investigational work." (Cockcroft, 1982:71) Clearly then "investigations" either as a group, a team or individually

will open opportunities for more fulfilling and demanding personal interactions between students in science and maths classes. Indeed objective truth and subjective meaning can now graze together.

Silver & Marshall (1990:284) report that having children work in cooperative groups to solve mathematics problems resulted in significant growths in children's problem-solving competence. They argue that in structured group settings, children would engage in better qualitative analyses and discover better 'initial problem representations' as a direct result of questioning in the group. They lamented that students in science and mathematics classes are rarely placed in such rich discussion environments. Philosophical discussions are such rich environments.

### PROVISIONS AND INNOVATIONS

Mathematics in particular is too often locked into skill reinforcement and application, and reformers are striving to move into the social and really useful areas like investigational work which "should start in response to pupils' questions, perhaps during exposition by the teacher or as a result of a piece of work which is in progress or has just been completed. The essential condition for work of this kind is that the teacher must be willing to pursue the matter ... The pupils should be encouraged to think [in hypothetical ways] and the teacher takes the opportunities which are presented by the members of the class. There should be a willingness on the part of the teacher to follow some false trails..." (Cockcroft, 250) Van Tassel-Baska (1992:51) also recommends applications of mathematics "from the real world through the creation of projects that provide that experience".

Hitchcock (1992) even advocates preparing and performing dramatic historical plays to show the excitement and drama of a mathematical discovery, "to tell the real story": "Within the iron dictates of scientific style ... a scientist cannot describe the excitement of discovery, the false leads, the hopes and disappointments, or even the path of thinking that may have led him through the various steps of his experiment" (22). He wants to make students aware of the human face of mathematics: "The struggles, victories, mistakes, disputes, the competition and the comradeship — but also the process of discovery, the rational reconstruction or dialectic of the story" (22).

So it is said the study of mathematics should show its human side: the competition, lust, pride, ambition, self-delusion, fear of the unknown, courage, endurance, the cry of victory, the fellowship of minds, the vanity and vitality. He says mathematics teachers need to replay the thrill of the hunt for truth: "Healthy children also manifest that primal conviction of the worth of the struggle, a zest and eagerness, a delight in learning and a desire to please, qualities that played their role in the birth of the Royal Society" (23). He suggests the students engage in dialogue and theatre to show both the human and the concept story. This process of learning through dialogue is very much the focus in this paper.

Judith Mousley (1992:294) takes up the same call and praises a learning process that celebrates "the false starts, the crises, the leaps and spurts of knowledge expansion, and the uncertain discoveries and rediscoveries of mathematical relationships." Philosophy in mathematics does enhance this process, the study of people articulating their ideas.

Four programming proposals for teaching philosophical inquiry in mathematics have been documented. Bradie and Duncan (1982) describe their undergraduate interdisciplinary course on ancient and classical cosmology and the Theory of Relativity. Their course about Zeno, Aristotle, Newton, Galileo and Einstein yielded "creative and new ideas about nature" (114). They found that students were able to resolve issues not traditionally tackled in science courses. They learnt how not to become over-technical, over-historical or over-philosophical and showed the greatly enhanced benefits of students learning subject matter from a double perspective.

MacDonald (1984) advocates teaching the formal inductive and deductive logic of the standard proofs at an early age. Brumbaugh (1984) demonstrates Plato's peculiar intuitive aesthetic symmetry in the discussion of justice and balance in *The Republic* with symmetrical ordinal numbers. This would seem to be suitable for senior maths classes. Finally Owens and Rottschaefter (1991) examine some significant philosophical questions about mathematics using the history of mathematics. Yet even these overarching courses still follow the traditional prescriptive methods; a more constructivist approach is advocated here.

Original materials for teaching philosophy on selected topics for Science classes can be readily generated: Items in logic from Lipman's *Harry*



(1976); the *Grid of Ignorance* (Kerwin 1983); a unit on the status of Observations, Measurement, Experts and 'Scientific' Knowledge; Beginnings and Possibilities; the Process of Scientific Discovery in *The Double Helix* (1978); and Ethical Questions in Science arising from *The Dragon in the Garden* (1978).

In detail, a typical Year 7 Australian Science syllabus covers: safety, reading scales, and measurement; changes of state, solutions, and mixtures; gravitational, kinetic and electric energy; the microscope and cells; classifications and keys; plants and seeds; the Earth, crust, structure and volcanoes; the planets; the five senses; temperature and its effects. (Source: Olsen M. and Olsen R *Science I* Rigby Australia).

Useful discussions in philosophy could be dovetailed in with these topics on the nature and reliability of our observations; the need for and reliance on expert observers; the weight and veracity of expert opinion; the nature, status and reliability of our knowledge; knowledge from the five senses; forensic and legal uses of sense knowledge; intermediaries, witnesses, experts and mediums in the process of learning; relativities of time and space; the physical properties of matter; problems of measurement and calibration; epistemic questions about point of view, aspect and definition in Science.

For instance, the *Grid of Ignorance* (Kerwin 1983) even looks like a science exercise. In this physical sense it is a focused task. Individuals could try it alone but the preferred big-picture class version requires group interactions, discussion and agreement. This demands the best in the community of inquiry method: respect, sharing, and raised self esteem.

The *Grid of Ignorance* came from American medical schools where it was successful in reminding students about their sources and limits of knowledge. I have given it limited trials in my (Australian) Year 8 classrooms and at a bright student's suggestion I have added the last category to complete the schema. I am sure it would be useful for all secondary levels at least. It raises many questions in kids' minds and the possibilities excite them. They thrill in organising such an intangible 'unthinkable' topic.

Philosophical issues in the process of a scientific discovery could be researched and discussed after a study of James D. Watson's *The Double Helix* Penguin 1978. One reviewer noted that "It demolishes the popular myth that scientific discovery is a process of discovery motivated by ideals

and directed at truth through the exercise of logic. ...[It celebrates] fallible people with fragile egos, luck, labour, ambition and intuition." D. Reanny *The Age Monthly Review* 1982.

Novels provide real contexts, motivations, circumstances and real life situations. They study ethical issues and describe life as it is lived. Novels have appeal and power as assigned reading for a course; they contextualise moral issues (Bowlden). Novels reveal and situate the darker side of ethics: treachery, cruelty, etc. They offer studies in contexts and tools. They facilitate traffic between philosophical arguments and student interests. They dramatise moral value, moral motivation and locus of action. (Jacobs)

Issues that arise in them include:

- creativity, intuition and excellence
- the place of mistakes in inquiry
- taking risks and scientific speculation
- inspiration and perspiration
- deduction via the process of elimination
- play as a productive process: "All we had to

do was to construct a set of molecular models and begin to play — with luck, the structure would be a helix." (p. 48)

Examining one novel, *The Dragon in the Garden*, in detail, philosophical topics could include:

**Issues:** Personal choices, one's duty to science, valuing knowledge, our duty to later generations, actions with permanent consequences.

**Text:** *The Dragon in the Garden* by Reginald Maddocks

**Activities:**

- Discuss the "finders keepers" theory.
- Discuss Jimmy's mental and emotional state: how he is confused, pressured, needing to make a mark, aware of his individuality, needing to make a difference to the world, etc.
- List his options as you see them in a friendly letter to him.
- Describe the whole situation from another's point of view: either Gladys's, Molly's, the local policeman's, his teacher's, or his parent's.
- Discuss the issues in the second last paragraph again more fully. Make some evaluations of them.
- As executive curator of the British Museum, make a reasoned and reasonable offer for the dragon.
- Perform a happy ending to this story starting from the ending of the extract.

- One student said that Jimmy felt this was his own, personal even sacred possession and that others seeing it would "defile" it in some way, would spoil it, take away its integrity "in its red-lined shape in a slab". Discuss this rather more subtle account of his feelings and how thus he justified his action to himself.
- What was Jimmy's duty to Science and humanity? Does this over-ride private property considerations?
- What does it mean to own something? Does an owner have the right to destroy that thing? Is ownership relative? What are the government's rights over mineral deposits in Australia?

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Philosophical learning activities appropriate to the science class could include:

Discuss the issues raised in reading.

Dramatise the inquiry: the frustrations, the steps, the deceptions, the successes.

Assess the discovery and compare it with other scientific discoveries.

Plot the scientific method used.

Make a model of the double helix.

Debate the topic: "Man will be destroyed by his own inventiveness."

## MAKING MEANINGS THROUGH DISCUSSION

Philosophy in the classroom focuses on language interactions. The community of inquiry is about dialogue, about interpersonal communication. Watson (1989:27) sees mastery of such communication as essential for empowering a new generation:

*Dialogue is essential for the development of mathematical thought, for visualisation of patterns and their interactions. In denying learners opportunities to work towards making mathematical meanings through dialogue we are denying them the opportunity to appropriate those genres of text which incorporate mathematical meanings.*

In fact the community of inquiry method builds up a relatively shared linguistic community and a community of values.

Students are empowered when they participate in the generation of the basic knowledge, in what

Watson calls "making mathematical meanings". Students are now being encouraged to reconstruct their own knowledge through experiences. The philosophical method offers students the chance to be creative in constructing their own meanings through dialogue.

Discovering and proving takes longer than just proving but it is far superior educationally. Cartwright et al (1985) outline their excellent proposal for such discovery through discussion:

*The act of participating in discussions forces students to communicate mathematically both verbalising their own (often partially formed) ideas, and reconstructing in their own words ideas that other people have proposed. By discussing problems among themselves, students often sort out each others' misunderstandings. ... By pooling their ideas, the group will often be able to find solutions to problems that no individual member of the group; could solve, with the result that each student will participate in solving more problems, and will see a greater variety of approaches to each problem than he could possibly do on his own. ... It increases their confidence in facing unfamiliar situations (pp. 14-15).*

The maths curriculum would be better for including these collective problem solving groups with community of inquiry methods: "... reconstruct in their own words ... sort out misunderstandings ... see a greater variety of approaches ... increase confidence in unfamiliar situations" Such a process is more holistic, striving for more relevance to the lives of the learners, fuller communication, and discovery learning for life. The community of inquiry method is a neat fit answer to these science and mathematics students' needs.

**THE COMMUNITY OF INQUIRY METHOD**

Philosophy in the community of inquiry method begins with student questions and students' interests as the starting points for inquiry; student concerns set its agenda for discussion. It is not teacher directed but centred on student interests. Hence philosophy in the classroom is a model educational practice for it offers a vital opportunity for recognising, validating and pursuing student concerns in a rigorous systematic way.

Newer constructivist methods now being advocated actually borrow from philosophy: "Teachers are encouraging children to investigate, discuss, question and verify. They are focusing on explorations and dialogues ... exposing them to the value and beauty of mathematics." (Standards, 1992) Individuals do construct their own knowledge bases and the rigour of philosophy will assist them.

The community of inquiry method in philosophy also broadens such skills bases. In *Standards*, the National Council of Teachers of Mathematics in the U.S.A. presents an exciting new vision with such phrases as "... organising and interpreting data, solving problems and making conjectures, validating rules and functions, exploring patterns." These verbs — exploring ... designing ... estimating ... patterning ... guessing — recur in the document and echo the proper tasks of philosophy.

The community of inquiry method constructs knowledge: "...when children construct or invent mathematics, discuss and write about their work, and solve a variety of problems ... they think mathematically, logically, visually and creatively. ... [Courses should] encourage students to introduce problems from their own experience ... to allow mathematics to come alive in the minds of the children" (Cruikshank and Sheffield, 1992:3). So studies done in this way become relevant to modern students, are creative, and are student-centred and student-driven.

**THINKING OUR WAY THROUGH THE CURRICULUM**

What can philosophy offer that a comprehensive maths course cannot? In reply, English & Cooper (1987) as teachers of mathematics pose this dilemma: for better problem solving, is it better to teach structured content or cognitive skills? They found that five thinking skills (visual thinking, logical thinking, patterning skills, flexible thinking and creative thinking) could be identified taught and measured. To meet specific needs, courses that teach thinking skills, strategies and executive plans within subject areas should be run on a short term basis as the need arises.

In a later article, English (1992) reports how Philosophy for Children can be integrated within the mathematics course proper, how her maths courses begin with students reflecting on themselves as mathematical thinkers then that they move on to consider the nature of thinking, think-

ing processes in problem solving and then the common barriers to successful thinking such as assumptions. I suggest that similarly philosophy in science classes could examine the modelling, explanations, contexts, contradictions and rules by which a conclusion, a law, a rule or a theory is reached.

Research shows that more successful problem solvers in physics are skilled at metacognition. Silver and Marshall (1990:278) report research that shows that successful students engaged in reflective thinking more often than did the less successful. It is a process of self-monitoring, regulation and evaluation of thinking. Experts, unlike novices, check, recheck and monitor. Philosophy teachers this habit of metacognition.

Philosophy brings these metaproceses to the surface. The steps of the scientific method, deduction, reasoning from analogy, the status of evidence, fact and opinion, systematic observation, possibility and probability are some critical areas usually taught only incidentally. But "teachers need specifically to focus on how they might go about improving their students' skills of thinking and dialogue" (Wilks, 1994:52). So science teachers can justifiably use philosophy to provide systematic methods for clarifying concepts, and for stimulus materials to broaden their students' learning activities.

Philosophy supports many of the skills proper to mathematics. Reed's Resource Book (1990) is an excellent accessible resource that takes up 'scientific' logic: matrix logic (37), analogies (41), hypothesising (55), inferences (58), and forecasting (60). Exercises to practise forced relationships, attribute listing and creative problem solving are simply presented there as prepared lessons plans.

Borasi (1992:192) recommends probability as a focus in a study of what constitutes a mathematical 'proof': "The study of probability in particular presents an additional advantage, since people's intuitions about chance and random events are often misleading and at odds with the results derived in probability theory, thus causing surprise and curiosity." To apply probability, students could read and discuss excerpts in Lipman's novel, *Harry* (Chapter 13, p. 67 line 7), and carry out exercises in the accompanying Manual (p. 333).

Secondly she recommends a study of 'infinity' to reveal how mathematicians as real people deal with uncertainty and approximation. Suitable philosophical discussions could be planned to dovetail in with mathematics sequences (see Appendix 1) when particular topics are being studied.

Thirdly, geometry is recommended as an appropriate topic for gifted mathematics students by Van Tassel-Baska (1992:51). Our argument is not that Philosophy will not replace these studies but that it can complement and enrich them.

Borasi (1992:190) outlines several novel strategies for practising such thinking skills in an humanitarian inquiry method:

- Exploit the complexity of real-life problematic situations;
- Focus on non traditional mathematical topics where uncertainty and limitations are most evident;
- Uncover humanistic elements within the traditional mathematics curriculum;
- Use errors as springboards for inquiry;
- Exploit the surprises elicited by working in new domains (anomalies);
- Create ambiguity and conflict by proposing alternatives to the status quo;
- Generate reading activities as a means of sustaining inquiry;
- Provide occasions for reflecting on the significance of one's inquiry.

These are excellent starters for philosophical discussion and inquiry. Attribute listing, hypothesising and evaluating are proper philosophical tasks. The potential of highlighted tensions and alternatives is very creative indeed. Using them, students learn skills through topics and take control of their own learning and practise that reflection on everyday life that is characteristic of a philosopher.

## NARRATIVES APPEAL

Everyone loves a story and in a story the story teller appraises his life-experience. Fictional stories extend the possibilities of human experience: true or not true, all narratives invite us to be on-lookers joining in the evaluation of some possibility of experience (Rosen, 1986:15).

To use this power of stories, Borasi (1992:194) points out that genuine stories meet genuine problems in an authentic context; the readers' interpretations may lead to alternative definitions of the problems themselves and possible alternative solutions; and they can familiarise students with the whole process of solving a mathematical problem; with them, readers and listeners take up a more active and critical stance; and finally, the affective elements in a story and its progress are also compelling learning foci.

It is no coincidence then that Matthew Lipman

chose story settings to situate his philosophical issues. His stories are inhabited by children with fears, hopes and joys that they can relate to easily. Lisa, Harry, Suki and the others live and move in settings that children can understand. Lipman's *Philosophy for Children* explores the basic reasoning tools — the techniques of critical thinking, formal and informal logic — which students in later years will be able to apply in subjects across the curriculum. It is a means to enhance the thinking skills of all students to give them control over their own learning processes.

Its classroom approach involves engaging the students in discussions of a variety of topics related to thinking in the stories: the process of inquiry, figuring things out, what a generalisation is, causes and effects, and so on. These issues arise in reading the Lipman contrived novels, Kio and Gus, Mark, Pixie, Harry, Lisa, and Suki where the child characters spend much time distinguishing better from poorer thinking. Passages are read aloud by the children who themselves choose the topics arising for discussion. These are recorded word for word to give the speakers "ownership" of their topics. Discussion of them then proceeds until satisfaction is reached. A comprehensive Manual for each novel offers checklists and exercises for enhancing these "leading ideas".

### CONCLUSIONS

This paper has reviewed theoretical and practical considerations for teachers of Science and Mathematics who are willing to extend and enrich their classes with Philosophy. It surveyed how both new and old content, processes and products can interact in programs teaching philosophy. It showed how appropriate issues can be selected from the Lipman novels, other published texts or even better from the children's own stories. It has advocated Borasi's humanistic inquiry approach in particular as a strategic way to foster the reflective and comprehensive thinking that Maths and Science students deserve. In this sense it argues that it is far better to be engaged in making meaning of 'facts' than to be just verifying their truth.

This paper indicated the power of stories; how writing and sharing stories and using narratives is a most powerful way to reexamine life's big issues. It argued for the community of inquiry method that facilitates a humanistic inquiry approach in science and maths. Its major purpose is to implement a rigorous philosophical inquiry which is

student-centred and student-driven, an inquiry to seek meaning and to discover relationships in our physical world.

The current TV advertisement declares that, "From day one, our lives are filled with numbers", but demonstrable priorities and articulated values must determine, shape and contextualise their meaning for us. Such meanings will be made only by young adults who are practised in philosophical thinking which empowers them to take effective control of their own destinies.

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### APPENDIX 1: Issues Selected Specifically for Maths and Science

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From Lipman, Matthew, Sharp, Ann Margaret & Oscanyan, Frederick S. (1984) *Philosophical Inquiry: Instructional Manual to accompany Harry Stottlemeier's Discovery* (2nd edition) Institute for the Advancement of Philosophy for Children, University Press of America.

| Topic                            | page    |
|----------------------------------|---------|
| Inference                        | 48-9    |
| Inductive reasoning              | 112-4   |
| Assumptions                      | 157-9   |
| Generalisations                  | 266-7   |
| Contradictions                   | 307-11  |
| Exceptions                       | 318     |
| Parts and wholes                 | 331     |
| The Four Possibilities           | 337     |
| Middle term                      | 377     |
| Causes and effects               | 395-400 |
| Explanations                     | 403     |
| Causes and reasons               | 405     |
| Hypotheses                       | 420     |
| Problem Solving by Inquiry       | 425     |
| Tautologies                      | 434     |
| Perspectives/frames of reference | 441     |
| Logic Review                     | 445-473 |

From Lipman, Matthew, Sharp, Ann Margaret. (1985) *Ethical Inquiry: Instructional Manual to accompany Lisa* (2nd edition) Institute for the Advancement of Philosophy for Children, University Press of America.

|                                   |          |
|-----------------------------------|----------|
| Wholes and parts                  | 6-8, 260 |
| Differences of degree and of kind | 19       |
| Supporting judgments with reasons | 26       |
| Hypothetical reasoning            | 39       |
| Consistency                       | 126      |
| Separating fact from opinion      | 132      |
| What is natural?                  | 225      |
| Underlying assumptions            | 289      |
| Quantitative measurement criteria | 325      |



|   |     |
|---|-----|
| Theories of truth                               | 356 |
| Discover or invent it?                          | 361 |
| Use of multiple criteria in<br>pattern analysis | 371 |
| Copernicus                                      | 381 |

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## APPENDIX 2: Sample Lesson: The Grid of Ignorance

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Through discussion, find examples and add comments to the grid:

### A MAP OF IGNORANCE

What we know

What we don't know

What we know we don't know

What we don't know we don't know

Apparent knowing

What we think we know

What we think we don't but we don't know that we do know

What we deny or refuse to look at

What we need to know

Source: Ann Kerwin (1983)  
adapted G. Smith & D. Joseph (1994)

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