A Primary School Curriculum to Foster Thinking About Mathematics

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Since the Fall of 1993, at the Centre Interdisciplinaire de Recherche sur l'Apprentissage et le Développement en Éducation (CIRADE) of the Université du Québec à Montréal (UQAM), two mathematicians (Louise Lafortune and Richard Pallascio) and one philosopher (Marie-France Daniel) have collaborated to design and develop a research project involving philosophy, mathematics and sciences. Previous observations in the classroom had led the researchers to realize that, within the school curriculum, children like some subject matters and dislike others. Most of them usually succeed in arts, physical education and language arts, but many have difficulties in succeeding in mathematics. Why? On the one hand, as Matthew Lipman advocates, the school curricula are not sufficiently "meaningful" for children (1980;1988). On the other hand, some studies in the field of mathematics suggest that there are myths and prejudices about mathematics in primary schools and that the school system is partly responsible for this. Indeed, the school system does not invite children to express emotions in class about mathematics nor does it favor creativity. It does not allow dialogue among peers about mathematical concepts and problems, nor the construction of mathematical knowledge by the students themselves (Lafortune, 1992).

For quite some time, myths and prejudices about teaching and learning mathematics have taken root. Some of these myths and prejudices are as follows: students have to toil and suffer to learn mathematics; every mathematical problem has only one correct answer; there exists one right way to solve a mathematical problem; inherent objectives of teaching and learning mathematics are found in speed and accuracy with computational skills; speed and accuracy are more readily achieved with competition than cooperation; there is no place for discussion in mathematics; logical and rational thinking are the main skills to foster in mathematics — not creativity and intuition; mathematics is very difficult and can be better understood by a few talented students; men and boys are more inclined to succeed in mathematics than women and girls, for males are rational and females are more intuitive and sensitive (Davidson, 1980; Lafortune, 1990).

Throughout history there have also been myths and prejudices about philosophy. Let us remember that in Plato's *Republic*, philosophy was a discipline reserved for a male elite, the rest of the community not being wise enough to deal appropriately with this double edge weapon (Book V). In the following 2000 years, philosophical thinking and philosophical discussions have often not been an expression of liberation which reveals the self, but a means of domination by language which shows to those who do not correspond to certain models, that they have good reasons to feel guilt, shame and fear. In this way, philosophy like mathematics has been a means of domination of certain individuals over others, often of men over women and children. Even today, at the end of the XXth century, philosophy is mostly restricted to higher education (college and university).

Some of the myths and prejudices which concern philosophy are: philosophy deals only with abstract concepts; philosophy uses particular idioms; philosophy is far from daily concerns; philosophy is dialectic and involves only logical and rational thinking; philosophy excludes intuition and feelings; philosophy always includes debates with effective rhetoric; philosophy is for those who possess mature thinking; philosophy is not for children (Daniel, on press).

In the face of all these myths and prejudices about mathematics and philosophy, the question arises as to whether there is anything university researchers and curricula designers can do about how students perceive and experience mathematics. We believe there is: based on a different way of thinking about mathematics, as well as a new way of doing philosophy, we seek to invite primary school students to participate in philosophico-mathematical communities of inquiry that will help them tame mathematics, understand better, like them better and have more pleasure in doing mathematics. In the following pages, we will present the philosophical foundations and epistemological principles inherent to the philosophico-mathematical curriculum we are designing and using in class. We will also include some excerpts of the material so far written. Finally we will present some qualitative results of experimentations held in three different primary schools.

THE PHILOSOPHICAL FOUNDATIONS AND THE EPISTEMOLOGICAL PRINCIPLES INHERENT TO THE PHILOSOPHICO-MATHEMATICAL CURRICULUM

We know that all educative activities, including mathematics, are processes which involve the aptitude to learn as well as the aptitude to teach. Children construct their ideas and attitudes towards mathematics and other subject matters by the means of ideas and attitudes that have been taught to them. This leads us to question various aspects of teaching and learning and to distinguish their roles in the process of apprenticeship.

a) To teach versus to learn: a conceptual distinction

In the constructivist and pragmatist points of view, to learn has to be differentiated from to teach. According to Latin etymology, to teach comes from "insignere" (to signal, to let others know) and to learn comes from "apprehendere" (to take with). The first term implies that the teachers are the subjects of the educative act, while the second implies that the students themselves are the subjects of the educative act.

To teach carries an ambiguous status: it is situated at the frontier of education and instruction (education being understood as an act from inside and instruction as having an outside cause). In the classroom, teaching is too often related to transmitting, which presupposes that students' role consist in receiving, memorizing and understanding rather than creating, inferring, evaluating (see Gilford in Paré, 1977).

To learn might also be ambiguous. In daily language, to learn might be used in its transitive form and it then involves a subjective act (to gain knowledge of something or acquire skill in some art and to become transformed by it). When used in its intransitive form, it involves an objective act (to become informed of, or about something by someone). Nevertheless, the Latin etymology of the verb "to learn" (apprehendere) reveals its essence, that is, to take with, to assimilate, to become able, to transform the self. Thus, to learn implies a voluntary and conscious act by a person to take the risk to get involved in the development of his or her capabilities (to do, to feel, to think and to be) in order to improve his or her comprehension of data, of self, of others and of life.

If to teach and to learn refer to two different aspects of education, we nevertheless believe that these two concepts are not conflicting because, in the classroom context, these concepts are complementary in nature. In the apprenticeship of mathematics for instance, we believe that there must be a part of teaching as well as a part of learning. Yet we believe that learning has a predominant role to play.

b) The first basic principle of learning: Learning is a process based on the reconstruction of knowledge by the self

What are the fundamental principles of significant learning? Pragmatists such as John Dewey would answer that a person learns through doubt and uncertainty. As Dewey points out, uncertainty brings about a process of discovery and learning (1916/1983; 1967). Ernest Bayles (1980), recapturing the Deweyan vision of learning, talks about the process of formulation of insights by the self as well as logical organization by the self. Constructivists for their part, state that the learning process starts with the self-appropriation of knowledge and with the construction of problems and their possible solutions by the students themselves (Bednarz et Garnier, 1989; St-Onge, 1992).

Following the pragmatist and constructivist perspectives and inspired by the pedagogy of Lipman and Sharp (1980), we have come to believe, concerning the learning of mathematics, that the teachers' role should focus on getting children involved in an active process of reconstruction of knowledge rather than giving a problem to students and ask them for the right answer. Indeed, to educate in mathematics should not merely involve enticing children to give final answers to problems. To educate in mathematics should privilege guidance of children involved in the process of mathematical inquiry (transformation, readjustment, reconstruction, improvement) (Daniel, in press b).

Some traditional pedagogists could ask: "What is there to inquire about by students in mathematics?" We answer that in this model, each step of the problem solving process could lead to an inquiry or to a construction: construction of the problems inherent to the mathematics problem; construction of the meaning of concepts inherent to a given problem; construction of the possible solutions to a problem; construction of the possibility of transfering the solutions to a problem to other problems and to various life experiences; construction of one's strength and character.

The regular application of this pedagogy should have a positive impact on children's motivation in doing mathematics and also on children's self-development, because construction by the self can foster, in time, self-development (Davidson 1980; Sharp, 1992). Indeed, to learn mathematics does not only mean to acquire knowledge and skills in mathematics but to learn how to improve one's ways of thinking, feeling, acting and being (Vincent, quoted by Brossard and Marsolais, 1992). This is confirmed by more and more researchers that show that learning mathematics is strongly related to attitudes and emotions (among others: Lafortune, 1992). Other researchers are trying to find ways to help teachers foster students' metacognition (Lafortune and St-Pierre, on press).

Many mathematics teachers and program designers contend, when it comes to teaching mathematics to students, that the teacher must privilege the cognitive aspects of learning rather than the affective or social dimensions (Baruk, 1994). We believe that such pedagogical and epistemological points of view leads to the persistence of many myths and prejudices about mathematics in primary school classrooms. Some of these myths and prejudices may be related to comments made by primary school children on mathematics: mathematics is useless in daily life; mathematics has no relationships with the rest of the curriculum; to succeed in mathematics, one has to find "tricks" and to think fast; mathematics is boring and hard; to succeed in mathematics, one has to be brilliant; the students who succeed well in mathematics are boring; students are not allowed to make mistakes in mathematics; it is a waste of time to try to understand mathematics; girls have to study more than boys to succeed in mathematics; mathematics teachers know everything (Lafortune, 1993).

In the face of such comments on mathematics, we believe it is about time to act towards changing attitudes towards mathematics and suggest the application of new pedagogical models in teaching mathematics. The model we have come to privilege is a constructive model (or some may say an inquiry model).

c) The second basic principle of learning: The intrinsic motivation to get involved

The second principle of significant learning in mathematics is called intrinsic motivation and is referred to in every book by John Dewey on education. For Dewey (1967), there exist two kinds of interests in an activity. The first kind is an interest that is generated by a person towards an activity (intrinsic) and is conducive to having the person succeed in the activity. The second kind of interest stems from an interest proposed by another person towards an activity (extrinsic) and is less conducive to success in the activity. With regard to the intrinsic interest and motivation to learn mathematics, Dewey states (1967) that as soon as studies in mathematics are dissociated from personal interest and their social utility, that is, when mathematics are presented as a mass of technical relationships and formulas, they become abstract and vain for students. It is only when children become intrinsically interested and conscious of mathematics as a means of solving daily problems (as opposed to ends in themselves), that they enjoy playing with numbers, symbols and formulas. Dewey recognizes the pedagogical and epistemological necessity to take into account the experience of students along with the role played by the self. He recognizes the affective aspect of learning.

Jean Piaget (1962) also lets us recognize the importance of personal interest in learning, by supporting the point of view that students who are interested in learning and are positively encouraged in the classroom, will have more enthusiasm to study and will learn more easily. He states that, for more than half of students, weakness in mathematics is due to affective blocks. Piaget contends that affectivity intervenes in the structures of intelligence, as a source of knowledge and of original cognitive acts (see also Daniel, 1992c).

For Mumme and Shepherd (1990), effective communication about mathematics enhances students' comprehension and empowers them as learners. In this sense, and considering the importance of "reflexive dialogue" in class, we believe that each and every student should have the opportunity, within the mathematics class, to share with the rest of the group, elements of his or her constructions regarding various problems. Indeed, there is no need for students to make efforts to answer the teacher's questions if there exists only one good answer and if everyone in the classroom has previously been asked to memorize it (Dewey, 1959). Children will be motivated to make efforts to solve mathematical problems, only if they know that their answers can make a difference and be useful to their peers (Bayles, 1980; Daniel 1992c). (One can also read: Lefebvre Pinard, M., 1989; Gelly, M., 1989; Blaye, A., 1989).

In order to respect the second principle of learning and foster students' interest in quality dialogue, the novels we are writing are philosophico-mathematical. The stories revolve around open ended mathematical concepts and problems (such as truth, proof, success, the infinity, figure versus shape) which call for discussion among children. We assume that if children realize that they have the right to propose different answers to such concepts and problems, they will quickly learn to enjoy doing mathematics. They will dissolve affective blocks towards mathematics and replace them by self-confidence and real interest and eventually produce better results in mathematics.

Also, it is fundamental that children come to realize that talent and success in mathematics do not proceed from innate dispositions, but rather from making good judgements. And children cannot succeed in making good mathematical judgements unless they continually practice making judgements. (Lipman, 1991; St-Onge, 1992).

It follows then, that children should have the opportunity to communicate and to work with each other in order to understand mathematical problems; that they should have the opportunity to identify the possible solutions to a problem and attempt to submit these solutions to concrete tests. It is through such dynamics that students will become responsible for their learning, that they will realize that they can learn according to their motivation to make efforts at participating to the elaboration of their own instruments of mathematical thinking (Daniel, 1992b; Lebuis and Daniel, 1993).

THE ROLE OF PHILOSOPHY IN THE DEVELOPMENT OF MATHEMATICAL LEARNING

Usually, students view mathematics as a demanding discipline, where only one answer is correct (McKnight et al., 1987). Discussions in mathematics class often lack the diversity of thought and originality we strive to develop (English, 1993). If learning as involvement of the whole self means anything in learning mathematics, we should privilege the development of reasoning, conceptualization, translating and researching in the mathematics class.

This brings us to the role of philosophy within the process of learning mathematics. First, let us specify what kind of philosophy is involved here. It is not the philosophy studied in traditional academic settings, but rather a practice of philosophy, a "doing" of philosophy which refers to Socrates' maïeutic (Lipman, in press). Doing philosophy to learn about mathematics involves the creation of a philosophico-mathematical community of inquiry where children practice at thinking about mathematics in an autonomous, critical and creative fashion. This community of inquiry is a locus where children can search for the meaning of philosophico-mathematical concepts; a locus where they can share their results with their peers in order to construct thinking about mathematics and contribute to their learning of different ways to deal with mathematics.

a) Conceptualizing

Most children in primary schools use and understand a limited form of language and address concepts in a limited fashion. For instance, if they often talk about truth, they rarely question mathematical truth; if they often ask for proof, they seldom ask for mathematical demonstration; if they often compare the number of stars to the infinite, they rarely talk about infinite numbers; if they often use the word number, they have difficulties understanding the distinction between number and numeral; if they know what a cube is, they do not know the difference between the shape of a cube and its sketch, and so on. This is to say, that mathematical language is formed of particular words whose meanings do not always correspond to those in daily language. We agree with Stella Baruk (Xerox copy), that we should not eliminate these words from students' books but, rather, help children understand the different meanings of these words, according to the different contexts they are used in. In this regard, Baruk wrote a dictionnary of mathematics (1992) to guide students in their search for meanings. We believe that a good way to stimulate this search, is to form a philosophico-mathematical community of inquiry in class where students are invited to clarify, together, the meaning of the mathematical words and concepts they are using very often without understanding them well. The community of inquiry enables students to practice at conceptualizing and at relating concepts to their different meanings, while at the same time, to practice at developing language skills through communication with peers.

Training in concept formation skills is meaningful for primary school students whenever it uses, as a starting point, the concepts usually used and understood by children in their daily language (Austin, 1970). Our philosophico-mathematical curriculum adopts this starting point by proposing to children, stories that depict daily situations. These stories address concepts such as truth, proof, infinite, too much, not enough, part of, and set the stage for children to talk about concepts in a community of inquiry. We believe that if children start to work with concepts that are meaningful to them, they will be interested and motivated to go further in their intellectual exploration and become authentic explorers in philosophico-mathematical language. As Lipman asserts (1991), if students work with concepts, they observe similarities and differences between two or more concepts, clarify ambiguities inherent to these concepts, establish and formulate relationships between them, explore their implications and imagine new contexts they might be applied to. In other words, in working with philosophico-mathematical concepts, children lean to think for themselves in the language of mathematics.

To think for oneself in the languages of the different subject matters involves critical and creative thinking, for autonomous thinking implies that a person is able to reflect impartially and objectively about others' discourse — as well as her's or his' — (critical). It also implies that a person is able to enrich this same discourse with fresh knowledge, new relationships and pertinent concepts (creative).

The fostering of critical mathematical thinking could have children realize they are not thinking by themselves when they are merely repeating a series of exercises. It could also have children become less prone to naive scientific creeds and less gullible in the face of pseudo-scientific authority claiming discourses of absolute truth. They could be less inclined to forget that most scientific discourses reflect hypotheses which have to be criticized, revised and modified (one can look at: Bednarz, Poirier et Bacon, 1992).

The fostering of creative mathematical thinking may help children create new useful concepts to better understand a theory; to discover a formerly unnoticed relation between two elements; to construct useful ordering; to organize the parts of a whole in a different fashion, and so on. According to David Tale (1991), some of the fundamental ingredients of mathematical creativity are relational understanding, intuition, imagination and inspiration. In short, because the fostering of concept-formation skills concerns philosophico-mathematical concepts, it represents more than a mere development of intellectual abilities. It is a global and fundamental education, which encourages students to think critically and creatively about mathematics and which gives them the possibility to articulate the expression of their opinions and contentions concerning personal as well as social or moral problems.

b) Reasoning

Reasoning and conceptualizing are strongly interrelated. Reasoning is the capacity of organizing different ideas into coherent systems often by means of human language. To do mathematics does not merely mean to get acquainted with the procedures of calculation (Baruk, 1994). To do mathematics is a way to imagine the world, to deal with reality, to reason about problems which are meaningful. When children, within a philosophical community of inquiry about mathematics, sit down and search together for the meanings of a mathematical problem, they develop their reasoning skills because, in order to succeed in their discursive activity, they have to extend the knowledge they already have (in regard with mathematics or with personal experience) through reasoning (Daniel, in press). When students search for meaning, they have to go through different proficiencies in such areas as classification, definition, question-formulation, giving examples and counter- examples, constructing and criticizing analogies, comparing, contrasting and so on (Lipman, 1991) — all proficiencies which are related to the development of reasoning.

c) Translating or Generalizing

To do philosophy about mathematics also involves and fosters translation skills. This is a high value skill, for to translate means to deal with human language. And as the reality of language is characterized by diversity and plurality, to translate means to deal with ambiguity. As with ambiguity, so with relationships. Actually, the most basic element of translation skills is found in relationships - in mathematics as well as in other disciplines. As Luis Radford (1992) states, a mathematics problem is never set down in vacuo: it always means a relationship to something. In this sense, to translate implies to establish meaningful relationships between one problem and another, between one solution and another, between one context and another, between one language and another.

Moreover, we are convinced that mathematical knowledge remains useless for students, unless they are able to transfer it to daily experience in order to improve its quality. Indeed, just as translating skills are fostered through philosophico-mathematical discussion, children will be able to construct their knowledge in other disciplines, to construct their vision of the world, to construct their own self. In this context, mathematics become a way of thinking and a means of communication.

As Michel St-Onge notices (1992), if the teachers first explain to children the solution to problems and then give them exercises related to the solution, children will never exercise translation skills and, consequently, when a real problem occurs, they will not know how to resolve it. Philosophico-mathematical discussions in the classroom exercise translation skills, for it gives students the opportunity to observe, test, construct and revise mathematical relationships. In this sense, translation is not only an intellectual act but a global behaviour: it recognizes the existence of a plurality of modes of reaching truth as well as the necessity to submit any truth to examination.

d) Inquiring

The last set of cognitive skills do not merely involve the articulation of questions but, also and mainly, the inquiring attitude which implies activities such as: observing, doubting, questioning, seeking reasons and searching for meaning (Daniel, 1992).

Very young children like to explore and always ask "Why?". But when they grow up they tend to look for clear-cut answers. They tend to put limitations to their inquiry by accepting (receiving) ready-made answers. And schools participate to this process by stressing the importance of clear-cut answers in addition to the accumulation and memorization of information. As Pallascio contends (1992), average teachers of mathematics will rarely put themselves in a situation of inquiry. If they do, they tend to avoid sharing the difficulties they encounter in inquiry with their students. They tend to hide the process of inquiry and only show the final term (see also Daniel, 1992b). Parallel to, it is rare that teachers propose to their students, real mathematical problems whose solutions are really unknown, a problem which is meaningful to students and which allows them to inquire, to invent and to reconstruct (see also St-Onge, 1992). Of course, stored knowledge is indispensable to question the results of research, to continue the exploration and to inquiry in general (Tall, 1991). Yet, to be fully educated in mathematics means to remain thirsty for new ideas and new questioning through the inquiry process.

In the philosophico-mathematical curriculum we are developing, students have to identify causes and effects, parts and wholes, means and ends and means and consequences, just as they have to suggest hypothesis, to formulate problems, to find solutions and so on. All these mental acts maintain and foster the inquiry attitude.

Summary

In working with a philosophico-mathematical curriculum, primary school students should train in the four varieties of cognitive skills (reasoning skills, concept formation skills, translation skills, inquiry skills). They should learn to communicate within a community of inquiry, develop affective and social skills, and eliminate, to one degree or another, some of the myths and prejudices related to mathematics.

THE PHILOSOPHICO-MATHEMATICAL CURRICULUM

The curriculum we are designing includes a philosophico-mathematical novel and a teacher's manual, in keeping with the tradition of Philosophy for Children. The novel depicts children's daily life experiences in relation to philosophico-mathematical concepts and problems. The manual is essentially composed of discussion plans about mathematical concepts and myths, philosophico-mathematical exercises and mathematical activities. This material will be used in mathematics classes from fourth to sixth grade. The main objective of the curriculum is to foster philosophical discussions among children with regard to mathematical concepts, problems, myths and prejudices.

a) Main Ideas

Some of the philosophico-mathematical ideas included in the material are:

- Can a room be a cube or does it only look like a cube?
- Do teachers know everything about geometry?
- Mathematics are useless, boring, difficult, and call for too much work.
- What is a problem?
- The fear of failing.

- Usefulness and uselessness.
- Too much and not enough.
- Abstract versus concrete.
- Is beauty in arts equivalent to beauty in mathematics?
- The necessity of proof and demonstration.
- Can animals think mathematically?
- Is geometry part of mathematics?
- Those who are good in mathematics rarely understand those who have difficulties in it.
- Where does success come from?
- The role of the community of inquiry in the finding of a solution to a problem of mathematics.
- Relationships.
- To guess and to reason.
- To believe.
- To understand mathematical operations and to memorize them.
- The role of intuition in mathematics.
- How can mathematics be useful in the resolution of daily problems?
- Infinite and indefinite.
- Does zero equal nothing?
- Rules, respect of the rules, exceptions to the rules.
- Does truth exist? Does mathematical truth exist?
- Do mathematics exist as an absolute, in the universe, or do they have to be created by human beings to exist?
- Far from; near of.
- Are mathematics a universal language?
- Number and numeral.
- To be a genius in mathematics.

Following the methodology of Philosophy for Children, students read one chapter of the novel by taking turns reading one sentence each. Then they are invited to ask meaningful questions brought about by reading the novel. Discussions on these questions take place within a philosophico-mathematical community of inquiry.

b) Extracts from the novel

Matilde enters her bedroom and slams the door behind her. She takes off her shoes, puts her pack-sack in a corner and throws herself onto her bed. Ah! How GOOD she feels!

Matilde enjoys her bedroom. It is a very small green room, with a square floor.
— Hum! it almost looks like a cube! Isabelle taught us something about the cube this morning, in the geometry class. What was she saying, exactly?

Isabelle's words come gradually to Matilde's mind. While she looks vaguely around her, Matilde wonders:
— Can a room really be a cube or can it only look like a cube? Isabelle told us that it was not possible to see, on earth, a PERFECT cube. this surprises me!

Matilde tries to think about this problem, but she is tired. She gets bogged down in her ideas; she becomes impatient and, finally, says to herself:
— Tomorrow, I will ask Isabelle to clarify this for me. After all, SHE is the teacher! She probably knows everything about geometry.

Matilde's thoughts fly away, released from their mathematical problem. She starts to daydream about her new boyfriend, Mathieu:
— Ah! Mathieu, what a guy!

Everything is now calm and pleasant in Mathilde's room when, suddenly, she sees a big red sphere passing in front of her. Her heart still beating, she recognizes her brother entering her bedroom and who has just thrown his basketball against the wall. What a pest!

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— I have a problem, Matilde.

— Really? Well, me too David, and it's YOU!

— No, please, listen to me. I really have a problem. I believe I've failed, once more, a mathematics exam this afternoon.

— Why do you say that, David?

— Because this is what I think, that's all!

— You said: "I BELIEVE I've failed, once more, a mathematics exam". What makes you believe you've failed? Is it your fear to fail or a prediction of failure? Maybe it is something else altogether.

— I don't really know. It is merely an impression.

— But, David, do you at least have good reasons to believe what you say? It is not because you have failed some tests last year, that you will fail them all this year.

— I know, but I hate mathematics!

— David, you always repeat to yourself: "I am not good in mathematics; students who are not good in mathematics fail their tests, so I will fail my tests". With this negative attitude, it is not surprising that you have failures.

— Mathematics are useless. The only thing they really do is provoke stress. And mathematics are so boring; they are difficult and call for too much work at home. I prefer to play ball or to draw. I am excellent in drawing!

After a moment of silence, David adds:
And that is what I am going to do. I will draw my "Frustrations" in my room. This will be useful.

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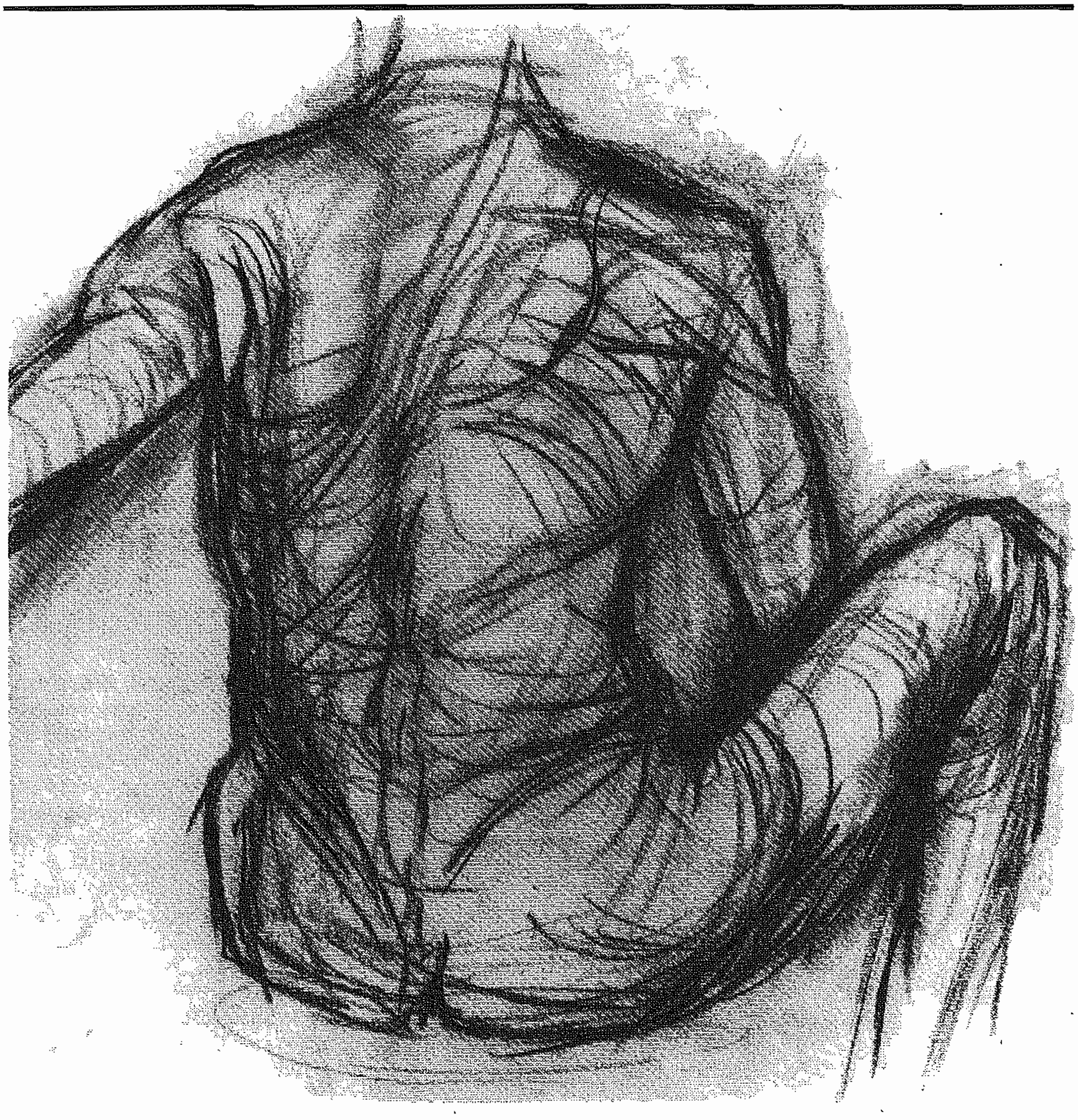
— Hi! Mathieu. Are you here for David?

— Yeah. Isn't he ready yet?

— No. He has to clean his room before leaving. Did you have a good time at your party last weekend?

— It was not bad, answers Mathieu. You should have come with your brother! he adds, blushing.



— What do you think? I heard you when you said you were embarrassed to invite me because I'm a "brain" in mathematics.

— It's not true! You're all making this up.

— I heard you perfectly well Mathieu! And if I heard you, it's true.

— Tell me, what is truth anyway? I think it's just a word that doesn't mean anything. Truth doesn't exist.

— How can you say such a thing Mathieu? There are many things that are true and on which we rely every day.

— Like what? Give me an example.

— Well like "The earth is round." Or "the earth revolves around the sun."

— But Matilde, don't you know that a few hundred years ago, everyone believed that the earth was flat?

— So?

— So what tells you we won't believe the earth is oval, a few hundred years from now?
— What's your point Mathieu?

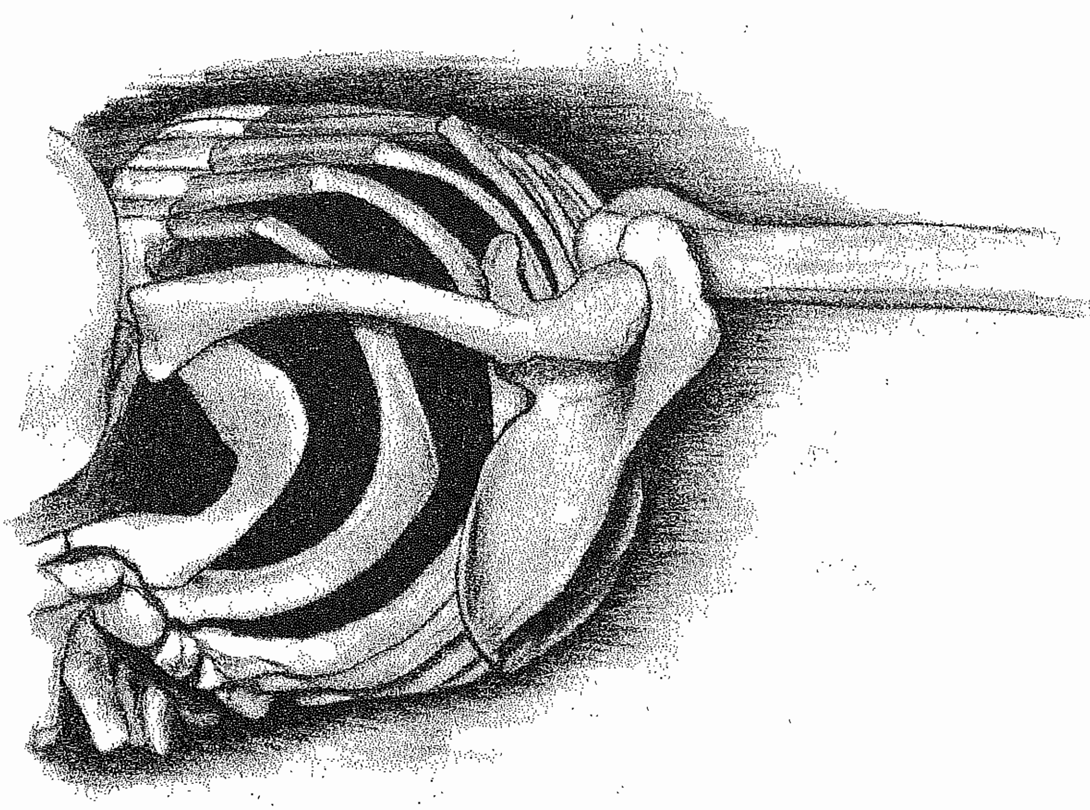
— What I'm trying to tell you is that what you say is true, that is, "The earth is round" may not be true. In the Middle Ages, the people saying "The earth is flat" were not saying something true.

— Matilde, you're saying the same thing as I am ! You're saying that something can be true for certain people and false for others. Like me, you're saying that truth doesn't exist.

— I'll give you another example. Let me think. Here we are: 2 + 2 = 4. This is always true and everyone agrees about it.

— I'm not so sure Matilde, that 2 and 2 have always been equal to 4 or will always be equal to 4.

— You're kidding!



— No I'm not. Close your eyes and imagine for a moment, that we're in year 2897.

— O.K. Go on.

— Picture it: strange beings are dressed differently and even look quite different when compared to us.

— What else?

— Keep thinking about these strange beings and try to get into their thoughts, now. Aren't they different from ours?

— Yes, probably.

— Continue your voyage in their brain, Matilde. Wouldn't you say their way of calculating is also different from ours?

— Maybe, I don't know.

— Well, I'm sure it is, Matilde. These beings are so different from us that they must have the need to invent a new way of calculating for them to evolve. I think that it's quite probable that in year 2897, 2 + 2 will equal 100 or something.

— I think mathematics are truths that can't change.

— Why? What makes you say that?

— I can't explain it to you, but I know it. I think mathematics exist regardless of what we may think of them.

— I don't get it, Matilde.

— Well, I think that mathematics are truths that already exist in the universe and that humans just need to discover them.

— I think quite the opposite: mathematicians have invented mathematics and since they are human beings, they can make mistakes or change their minds. So for me, there is no mathematical truth.

At this point, David, who was listening in, asks:
— Tell me Mathieu, how and why would humans invent something like mathematics?

— To progress! replies Mathieu.

— Nah, says David. I think that mathematics exist in the universe the same way the stars out there exist. It's the astronomer's or the mathematician's job to discover them.

At this point, Matilde's and David's mother comes in the kitchen intrigued by the conversation she was overhearing from the living-room.

Looking at Matilde and David she says:
— Let's say that mathematics already exist in the universe. Would that mean that mathematicians have never created or invented a mathematics formula? To me that would be impossible.

Before Matilde and David could react, she turned to Mathieu:
— Let's say that mathematics exist in the minds of humans, would that mean that a baby would, at birth, possess the ability to calculate? Wouldn't it be strange?

Mathieu was surprised by his friends' mother's question and since he's shy, he chooses to run off:
— I'm sorry Mam, but we have to go. It's already late. Right David?

Back in her room, Matilde whispers:
— I still wonder if truth exists.

SOME REACTIONS FROM AN EXPERIMENTATION IN THREE CLASSROOMS

Since February 1994, a qualitative experimentation of the philosophico-mathematical material is carried out in classrooms of three different primary schools. After each class, teachers have to fill out an evaluation form (concerning the novel, the manual, and the discussion). Moreover, each discussion is recorded on audio tape.

To this date, we cannot provide an analysis of the discussions. Nevertheless, based on the evaluation forms filled by teachers, we can share the following comments:

— The students are glad to see that David does not like mathematics.

— The students find the novel more interesting and easier to read than what they trained on in Philosophy for Children.

— Children are very helpful in suggesting ways to make discussions more interesting.

— There is a high participation of children to discussions, although relating discussion contents to mathematics is not always fun for some of them. Sometimes they don't want to hear about mathematics.

— Exercises and activities proposed by the manual are useful.

— Sometimes, it is difficult to establish a direct link between the topic of the discussion proposed by the students and the choice of exercise or activity proposed in the manual. Sometimes, the teachers mention, we are left to adapt or invent, on the spot, an exercise or an activity clearly related to the discussion. It is difficult to succeed at this.

— We should get into more practical activities related to mathematics to help us talk about them.

CONCLUSION

We believe that to design and apply a curriculum which would foster the philosophical dialogue about mathematics is a significant way to start to tame and to like to learn mathematics. Indeed, we believe that if children do not like mathematics it is because they hardly see their relationship to the daily world or to their own personal problems. A philosophical curriculum has the power to help children establish this relationship, for it is:

1. A tool adapted to children, which talks to them in their own language and about their own difficulties and interests in regard with mathematics;
2. A tool that can foster thinking about mathematics, because philosophy contains universal concepts which can be dealt with by children as well as by adults.

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